Compressive Wireless Sensing in Internet of Thing： key technology and application

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- Sparse Signal Transmission via Lossy Link Using Compressive Sensing
- Sensors, accept

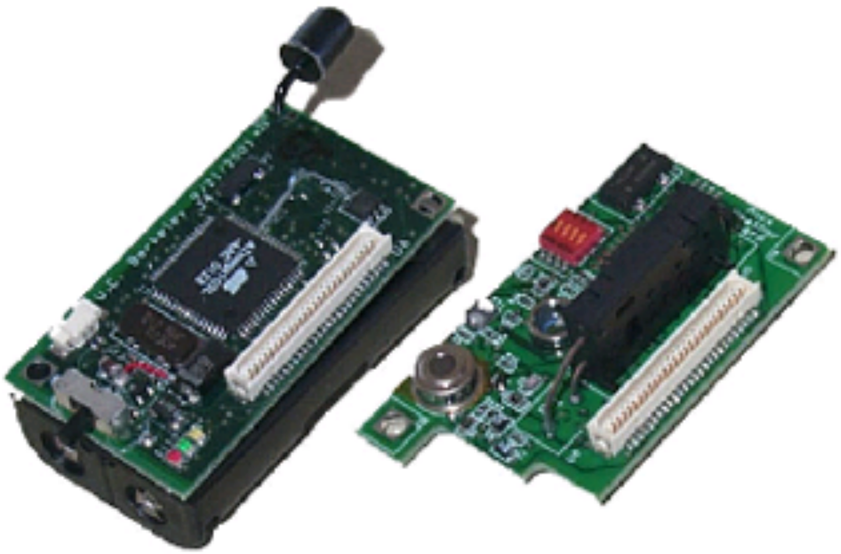
- DoA Estimation from compressed Wireless Array Data via Joint Sparse Representation
- *IEEE Trans on Signal Processing* (**IEEE TSP**) , in second review

Wireless, Ad Hoc Sensor Network

- Smart sensors
  - Transducers
  - Power
  - On-board processor, storage
  - Wireless transceivers
- Ad hoc network
  - No predefined, fixed network configuration
  - Transmit, receive, and relay information
- Wireless communication
  - Radio, infrared, optical, and other modalities

- Vision
  - Smart environment:
    - Monitoring
    - Control, interaction
  - Large number of low cost sensor nodes deploy-n-play, self-configuration to form network, Collaborative in-situ information processing
- Applications
  - Environmental monitoring
  - Civil structure/earth quake monitoring
  - Premises security
  - Machine instrument diagnosis
  - Health care

Mica Sensor Node

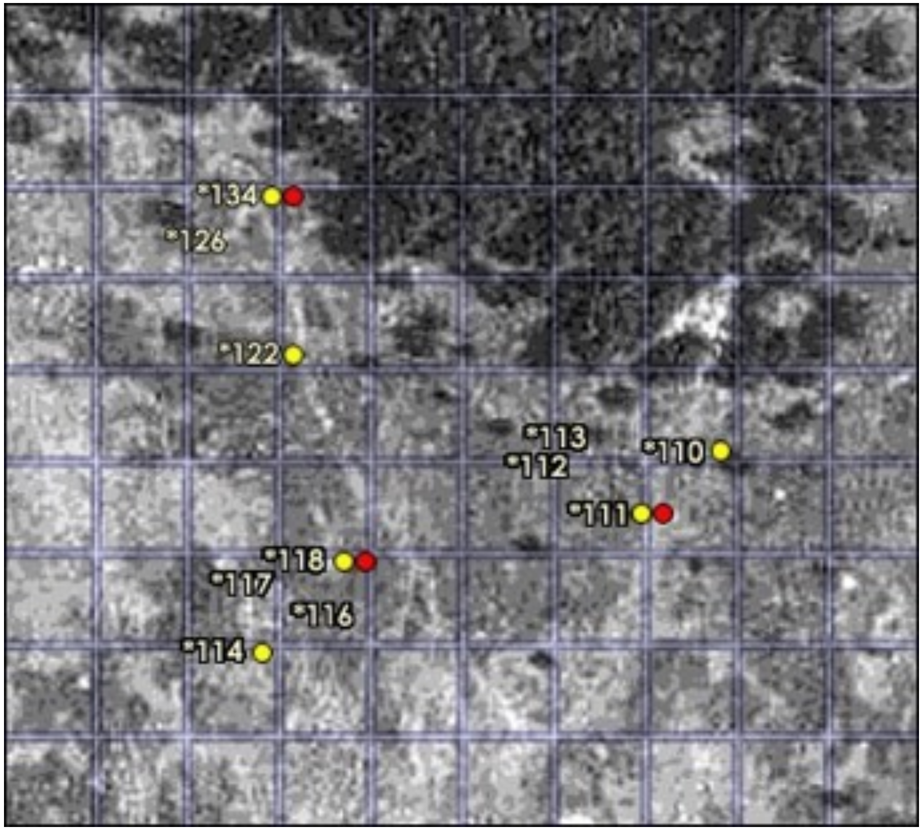


Left: Mica II sensor node 2.0x1.5x0.5 cu. In.

Right: weather board with temperature, thermopile (passive IR), humidity, light, acclerometer sensors, connected to Mica II node

- Single channel, 916 Mhz radio for bi-directional radio @40kps
- 4MHz micro-controller
- 512KB flash RAM
- 2 AA batteries (~2.5Ah), DC boost converter (maintain voltage)
- Sensors are pre-calibrated (±1-3%) and interchangeable

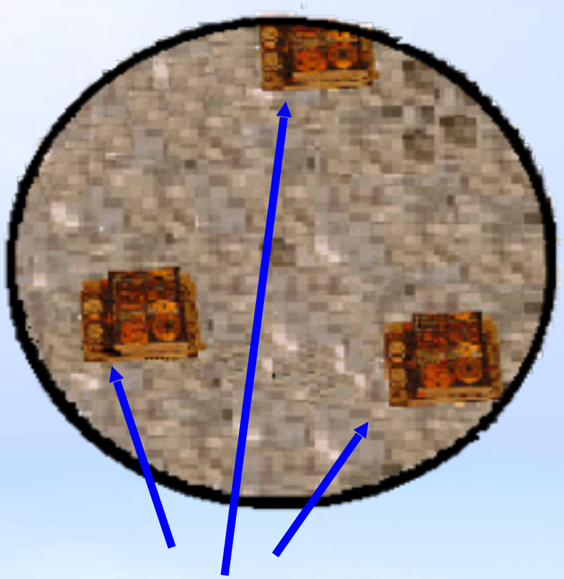
Habitat and the Bird



Habitat to be monitored (up, yellow: microphone Red: camera) and the Leach's storm petrel (right)

Environmental monitoring

The inside wall of drainpipe



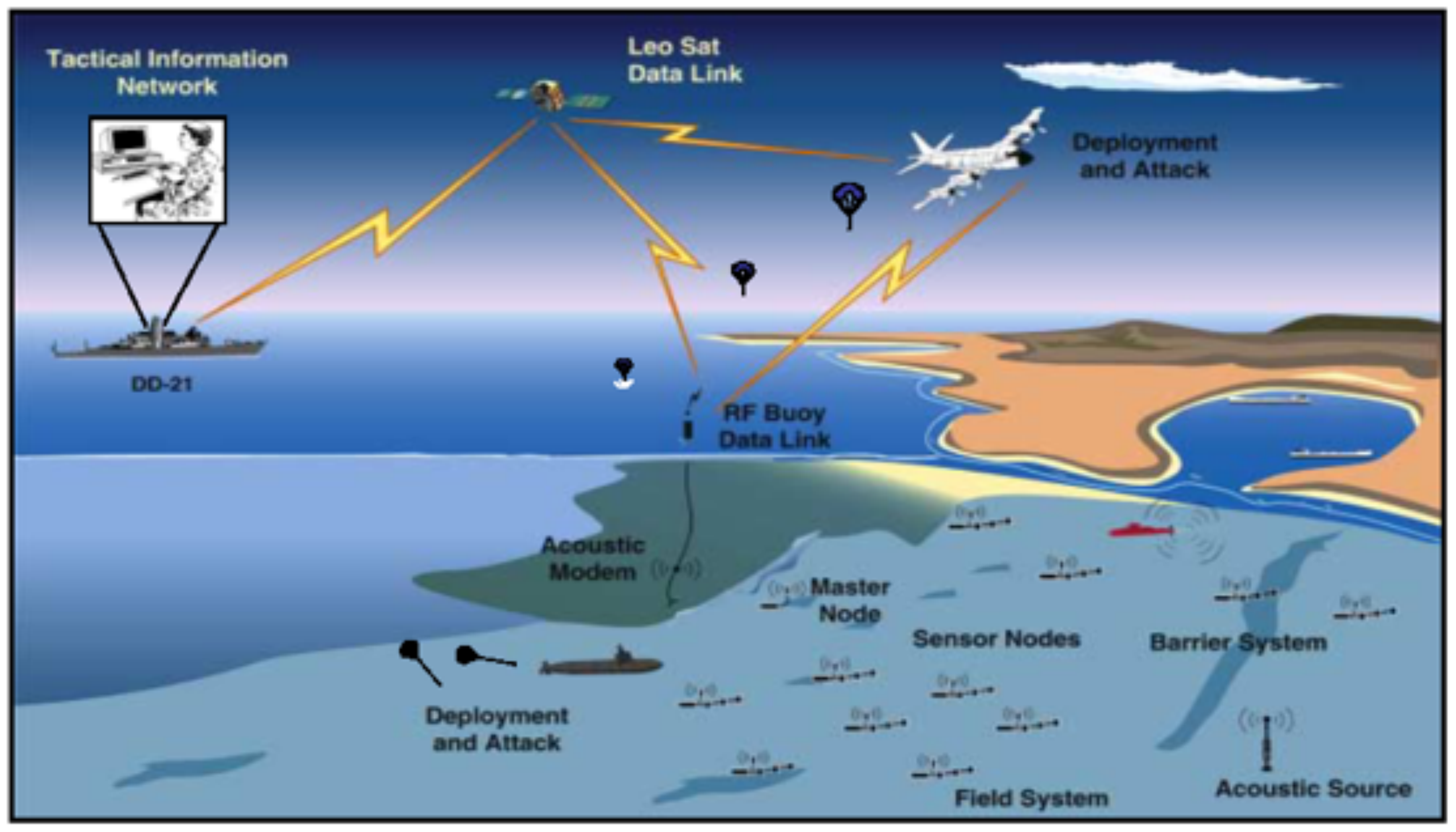
Sensor nodes

Pollution monitoring

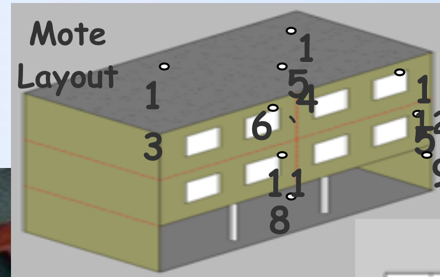
Great Duck Island Monitoring Project

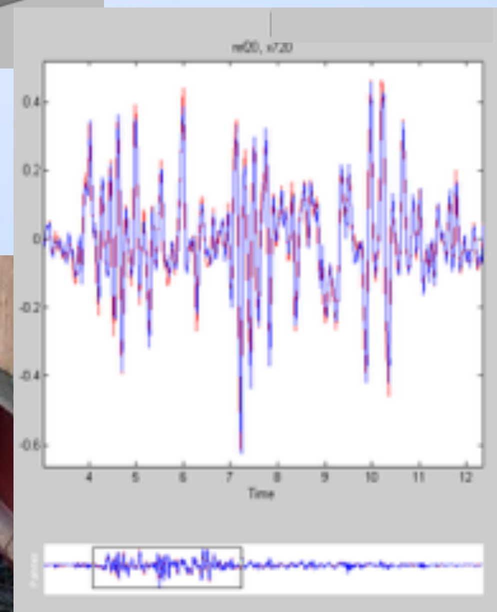
- **Mission:**
  - monitor the microclimates in and around nesting burrows used by the Leach's Storm Petrel.
- **Goal:**
  - to develop a habitat monitoring kit that enables researchers worldwide to engage in the non-intrusive and non-disruptive monitoring of sensitive wildlife and habitats

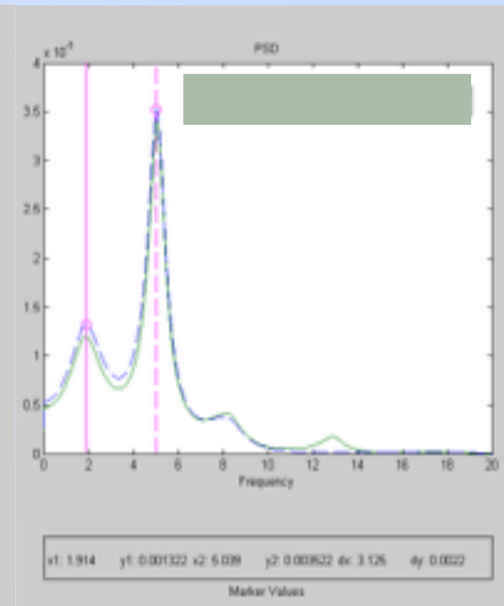
Military Surveillance



Structural Monitoring

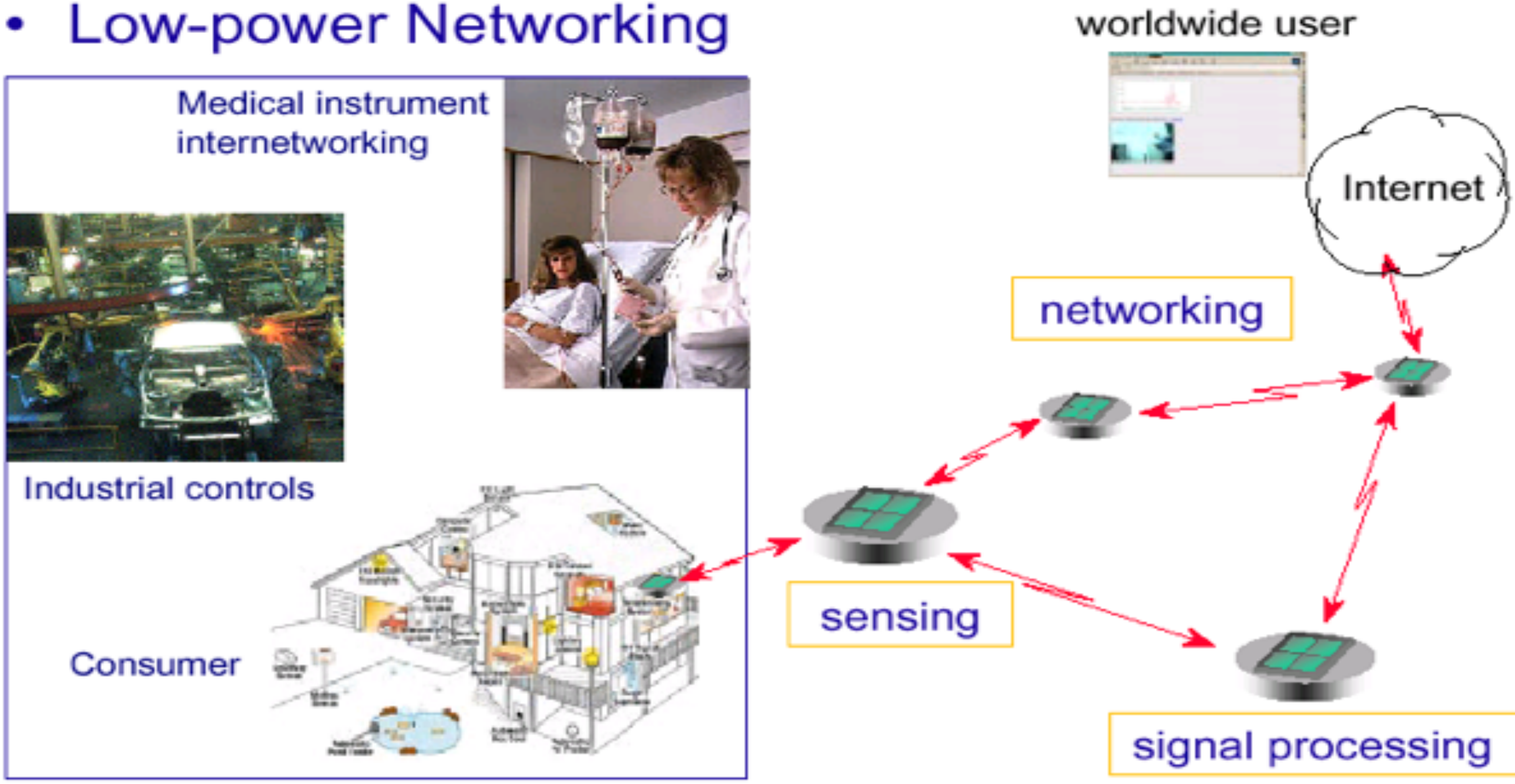






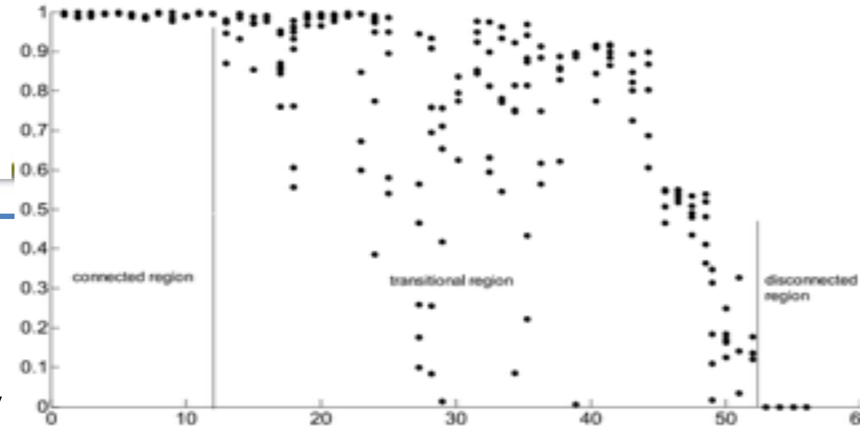
Other Applications

- Low-power Networking



Other Applications

•Existing problems



Distance between sender and receive
(P8)

Resource-constrained WSNs (IOT)
Constrained Power
Constrained Computation Capability

Low power wireless communication
Larger volume data to transmit （Nyquist Sampling）
Unreliable communication due to lossy link

Waste and un-proper utilization of WSN
•Error correction (Channel coding and ARQ) relies on Sender
•Communication bandwidth waste due to no use of lossy link

Part1 — Background and Motivation

- Introduction of compressed sensing

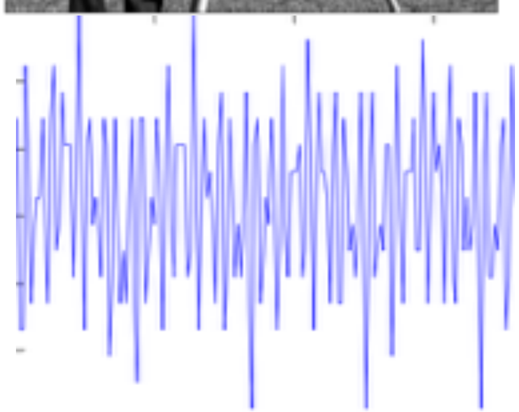
Many signals can be ***compressed*** in some basis (Fourier .etc.)

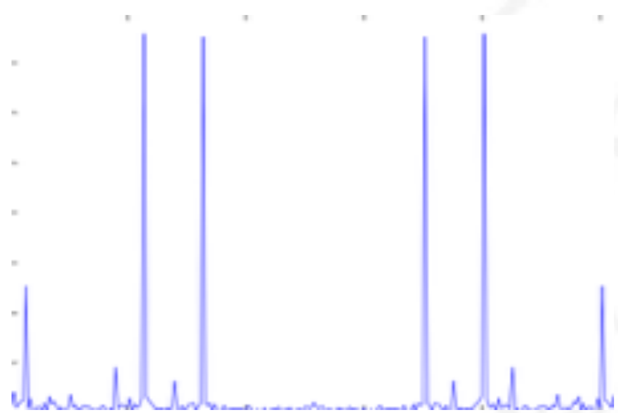
Ubiquity of sparse signal in WSNs

N
pixels

$K \ll N$
large
wavelet
coefficients

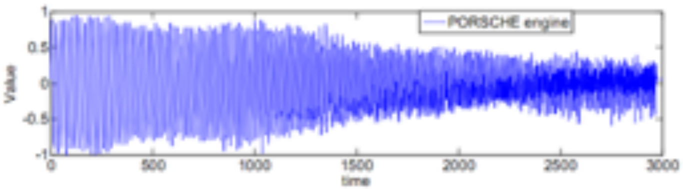
N
acoustic
signal
samples



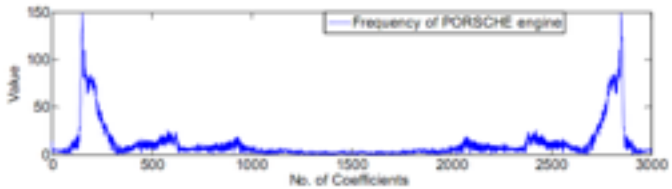


$K \ll N$
large
Fourier
coefficients

Sparse Signal Transmission

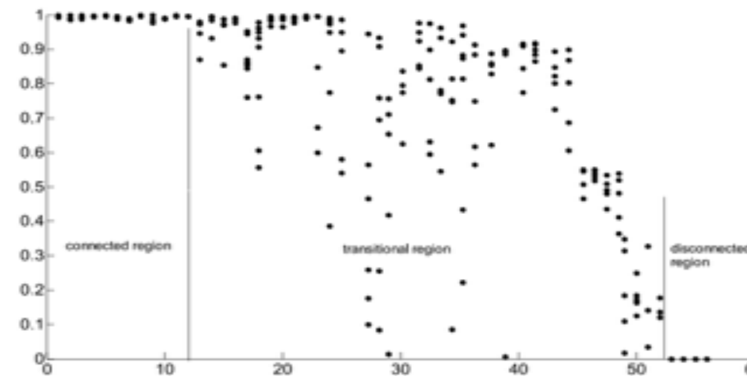


Original signal — Time domain



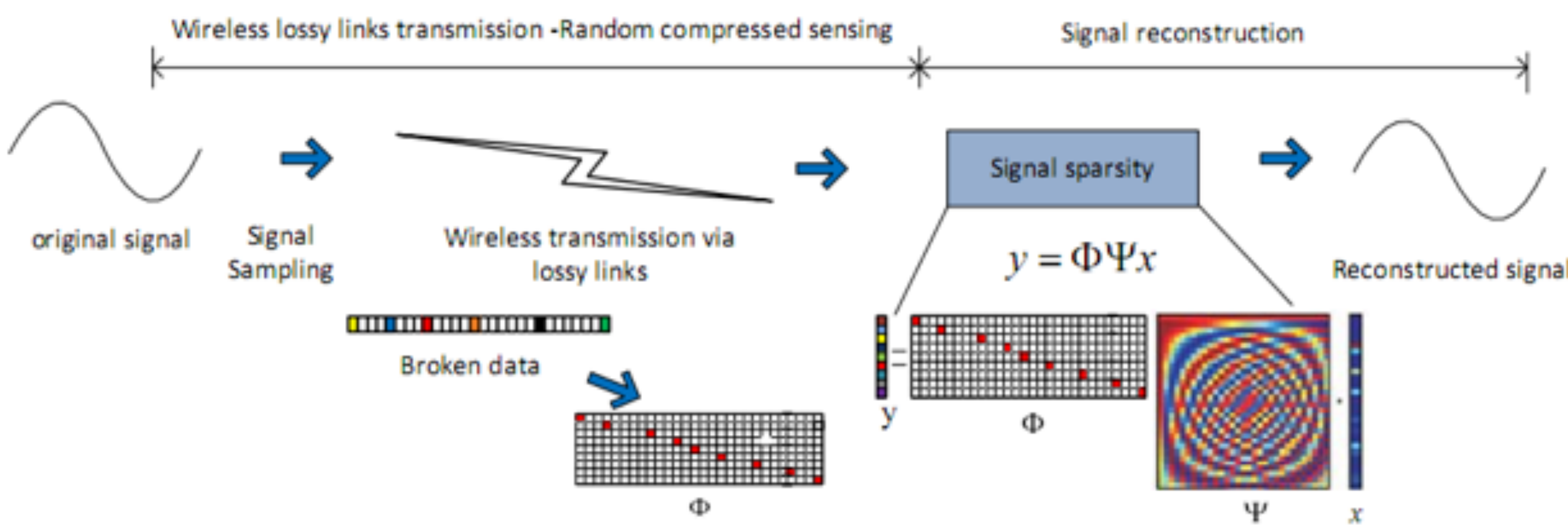
sparse signal — Frequency domain

PRR



Distance between sender and receiver

Process



Part1 — Background and Motivation

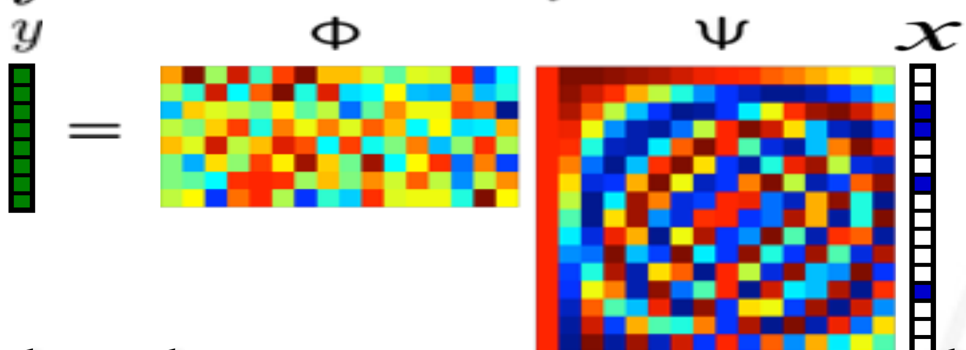
- **Compressive sensing fundamentals**

Sparsity representation

$$f = \Psi x \ , \quad f, x \in R^N \quad \| x \|_0 = k << N$$

Projection matrix construction

$$y = \Phi f = \Phi \Psi x \ , \quad y \in R^M \quad M << N$$



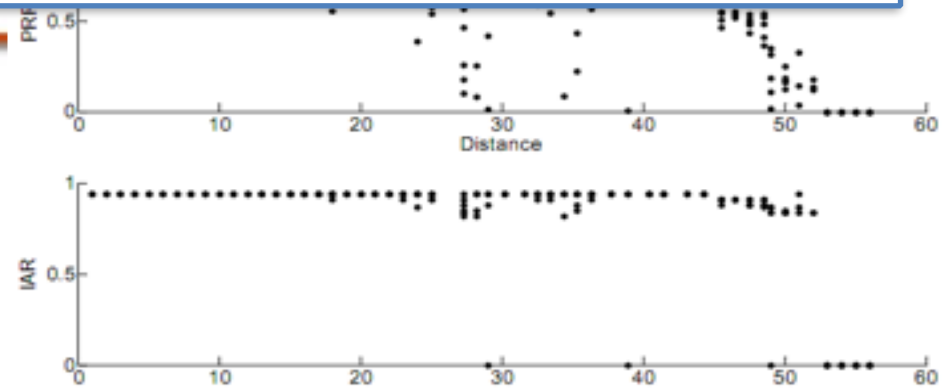
Data is local, measurements are global!

Reconstruction algorithm

$$x = \arg\min \| x \|_0 \quad s.t. \ \Phi \Psi x = y$$

$$M \geq ck \log n$$

Existing problems and opportunity



Distance between sender and receiver

Efficient use of lossy link
Expand communication range

Resource-constrained WSNs

- Constrained Power
- Constrained Computation

CS-based wireless communication systems in WSNs

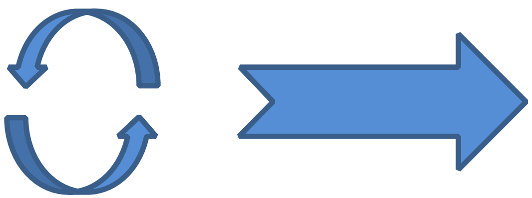
- Smaller volume data (Compressive Sampling)
- Efficient use of lossy link without expensive
- channel coding and ARQ
- Avoid High cost of high speed sample
- Shift the burden to Receiver

$$M << N$$

Part1—Background and Motivation

•Existing problems

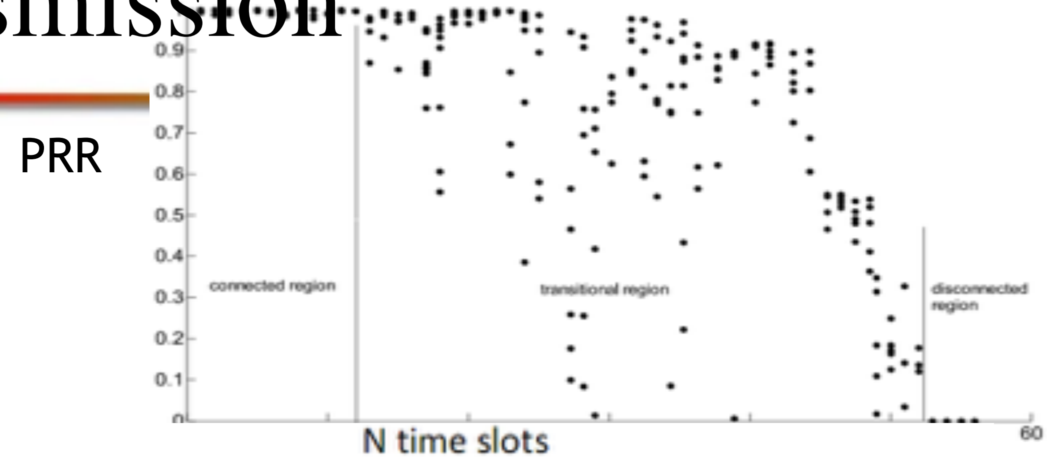
Resource-constrained WSNs
•Constrained Power
•Constrained Computation capability



Promote Efficiency、lifetime of resource-constrained IoT

CS-based wireless communication systems in WSNs
•Smaller volume data (Compressive Sampling)
•Efficient use of **lossy link** without expensive channel coding and
•High cost of high speed sample
•Shift the burden to Receiver

Sparse Signal Transmission



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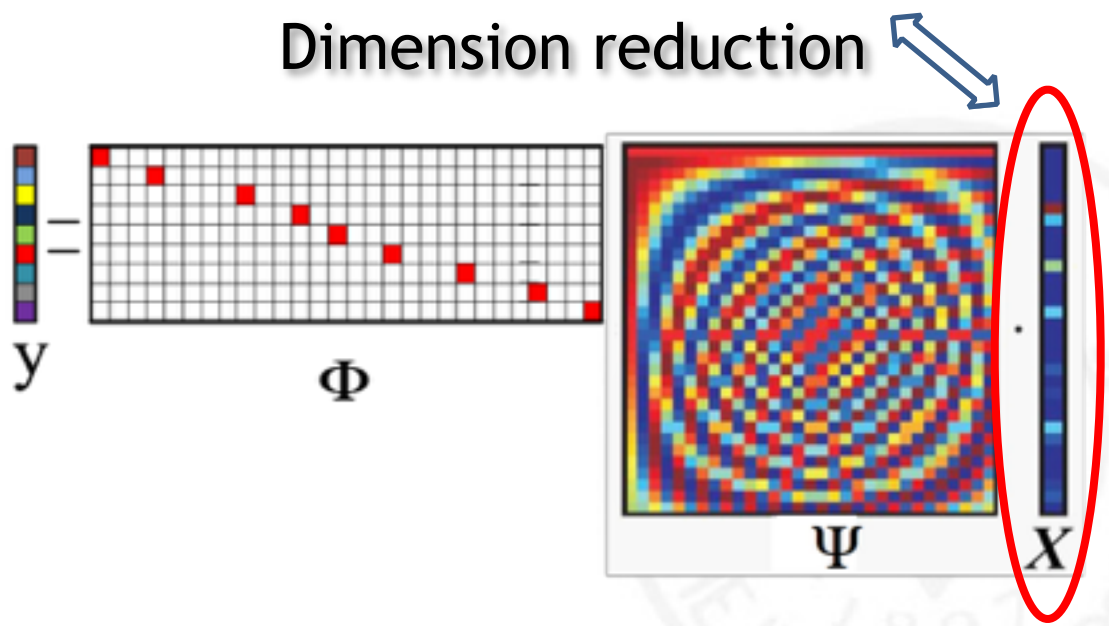
Random data loss

Dimension reduction

Random compressive sampling

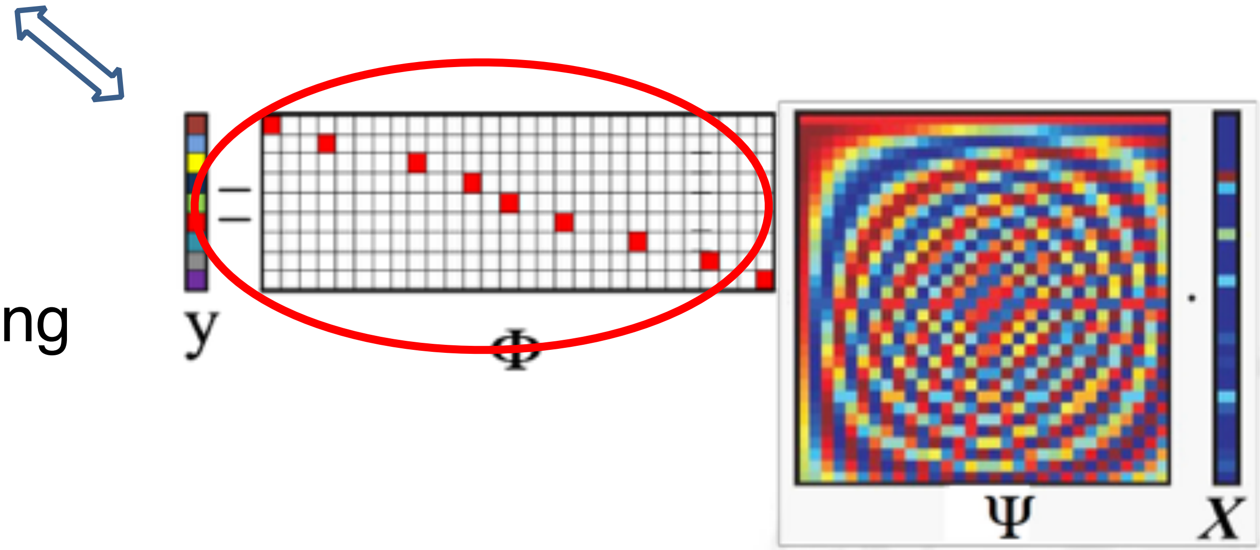


Sparse Signal Transmission

- Easy-to-implement projection matrix

$$\Phi(i,j) = \begin{cases} 1 \ \text{if} \ j = J(i) \le N \\ 0 \quad \text{otherwise} \end{cases}$$

Random compressive sampling



i is the row of projection matrix Φ, also the received packet sequence number . $J(i)$ is the original sequence number in f

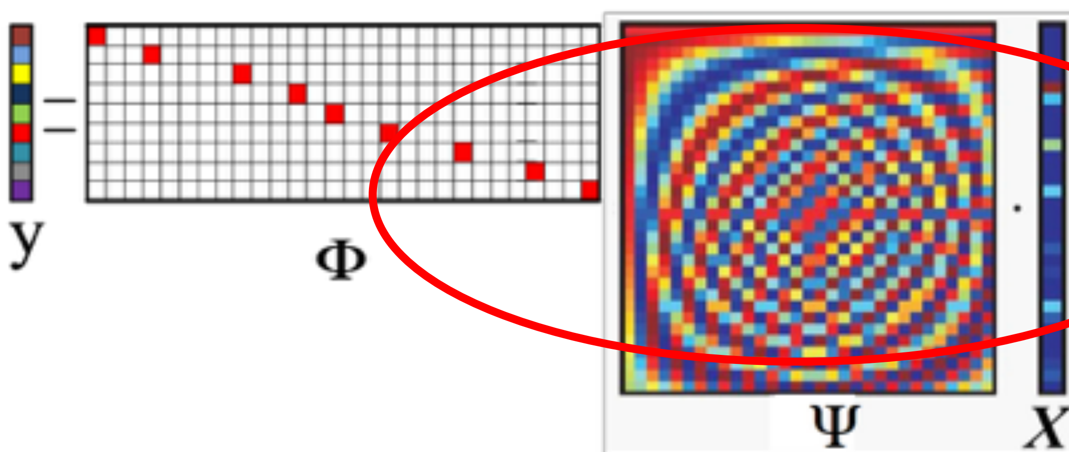
Sparse Signal Transmission

- Easy-to-implement projection matrix

$$\Phi(i,j)=\begin{cases}1 \ if \ j=J(i)\le N\\ 0 \qquad otherwise\end{cases}$$

Partial Fourier Basis

Random compressive sampling



i is the row of projection matrix Φ , also the received acket sequence number. $J(i)$ is the original sequence umber in f

Sparse Signal Transmission

CS effect on wireless link

Original signal

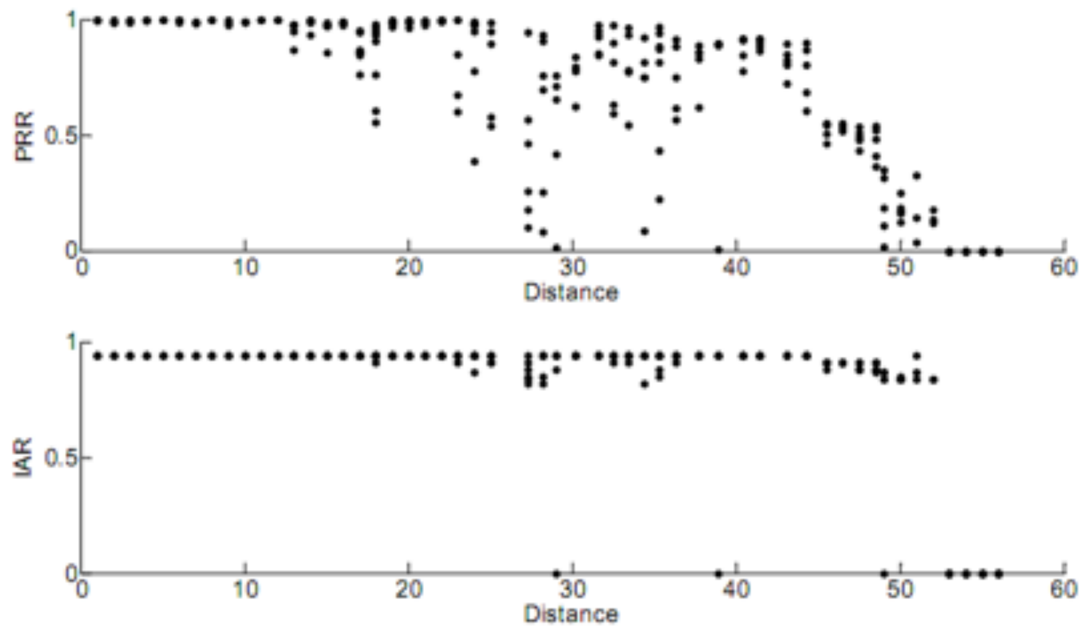
$$x = (x_1, x_2, \ldots, x_N)$$

Reconstructed signal

$$x' = (x_1', x_2', \ldots, x_N')$$

Reconstruction error

$$\sigma = \frac{\sqrt{\sum_{i=1}^{i=N} (x_i - x_i')^2}}{\sqrt{\sum_{i=1}^{i=N} x_i^2}}$$



- Expand communication range

IAR

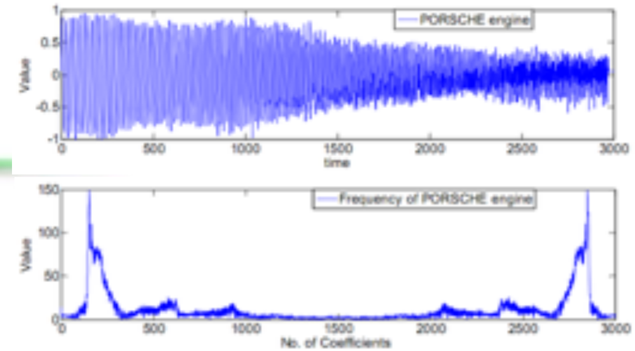
Information acquisition rate

$$IAR = \begin{cases} 1 & 0 < \varepsilon < threshold \\ 0 & \varepsilon > threshold \end{cases}$$

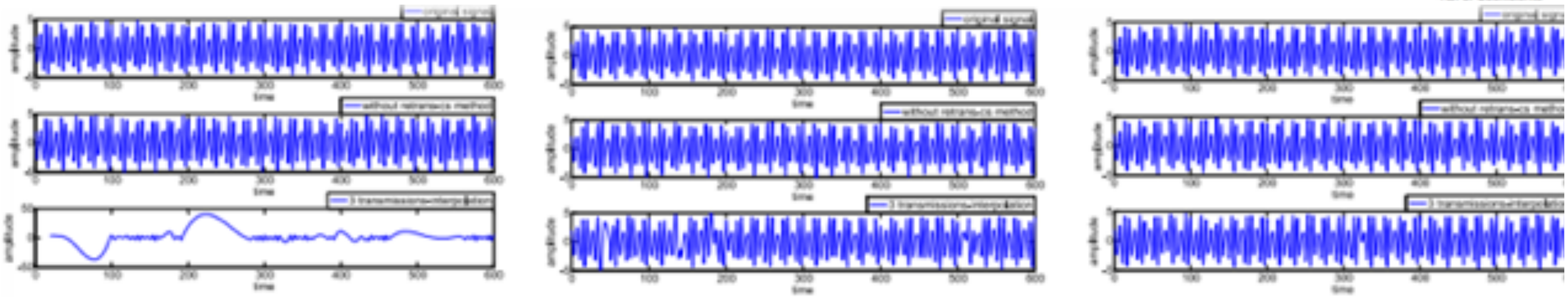
$threshold$

denotes the application's need for signal recovery error

Sparse Signal Transmission



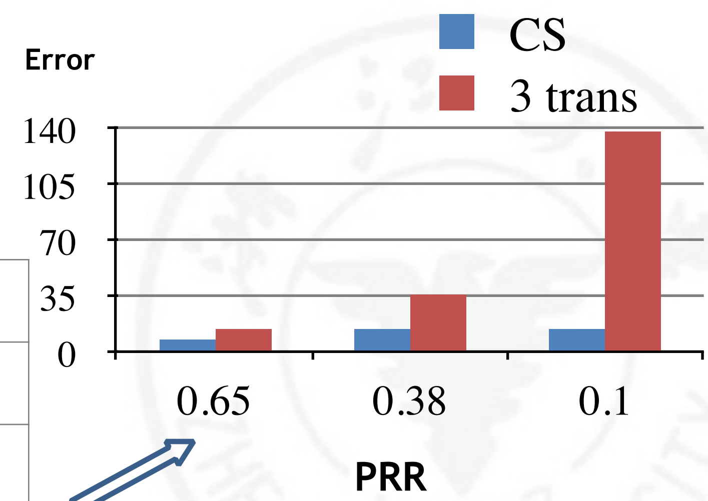
- CS effect on sparse signal transmission



a) Reconstruction comparison with single ransmission PRR=0.1 (b) Reconstruction comparison with single transmission PRR=0.38 (c) Reconstruction comparison with sin transmission PRR=0.65

Experimental parameter and results

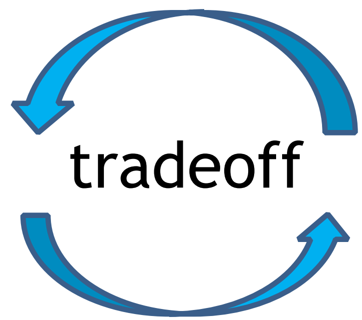
PRR	CS/M	CS/Error(%)	3 trans/M	3 trans/Error(%)
0.1	60	14.83	161	137.35
0.38	228	13.7	455	35.72
0.65	392	8.35	569	14.26



Packet length control

Data packet structure of IEEE 802.15.4

Longer packet
overhead ↓



Shorter packet
error rate ↓

Packet length control is an easy-implement and efficient method to promote communication performance.

Packet length control

Data packet structure of IEEE 802.15.4

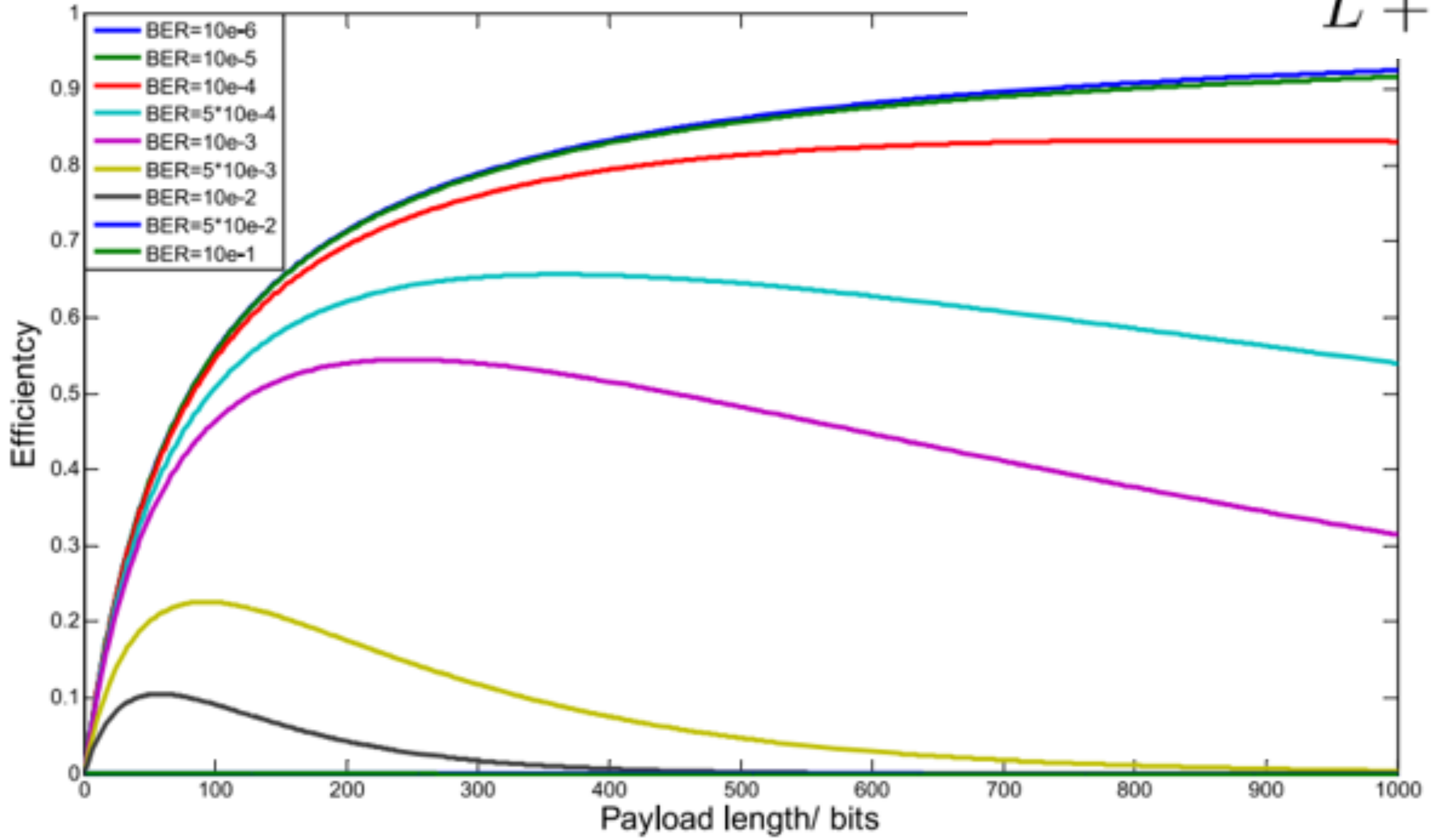
- L= length of payload
- O= Length of overhead (MHR and MFR)
- BER= Probability of channel bit error; a function of transmitter's power and path loss
- E= Normalized data transmission efficiency

$$E = \frac{L * (1 - BER)^{L+O}}{L + O}$$

Lettieri P, Srivastava M B, *Adaptive frame length control for improving wireless link throughput, range, and energy efficiency*, Seventeenth Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM'98), pp. 564-571, 1998.

Packet length control under traditional method

$$E = \frac{L * (1 - BER)^{L+O}}{L + O}$$



Data transmission efficiency vs payload length under varying BER

Packet length control under CS

Data transmission efficiency

$$E = \frac{L * (1 - BER)^{L+O}}{L + O}$$

Signal transmission efficiency ?

How to measure *signal* transmission efficiency?

error $\varepsilon = \frac{\sqrt{\sum_i (x_i - \hat{x}_i)^2}}{\sqrt{\sum_i {x_i}^2}}$

Objective $\min ||x - \hat{x}||_2$

Subject to $y = (\hat{x}_1, \hat{x}_2 ... \hat{x}_N)$

original signal $\mathbf{x} = (x_1, x_2 ... x_N)$

Packet length control under CS

Packet length vs recovery error vs mutual coherence

$$\mu(\mathbf{A}) = \max_{i \neq j, 1 \leq i,j \leq N} \left\{ \frac{|\mathbf{A}_i{}^T \mathbf{A}_j|}{||\mathbf{A}_i||.||\mathbf{A}_j||} \right\}$$

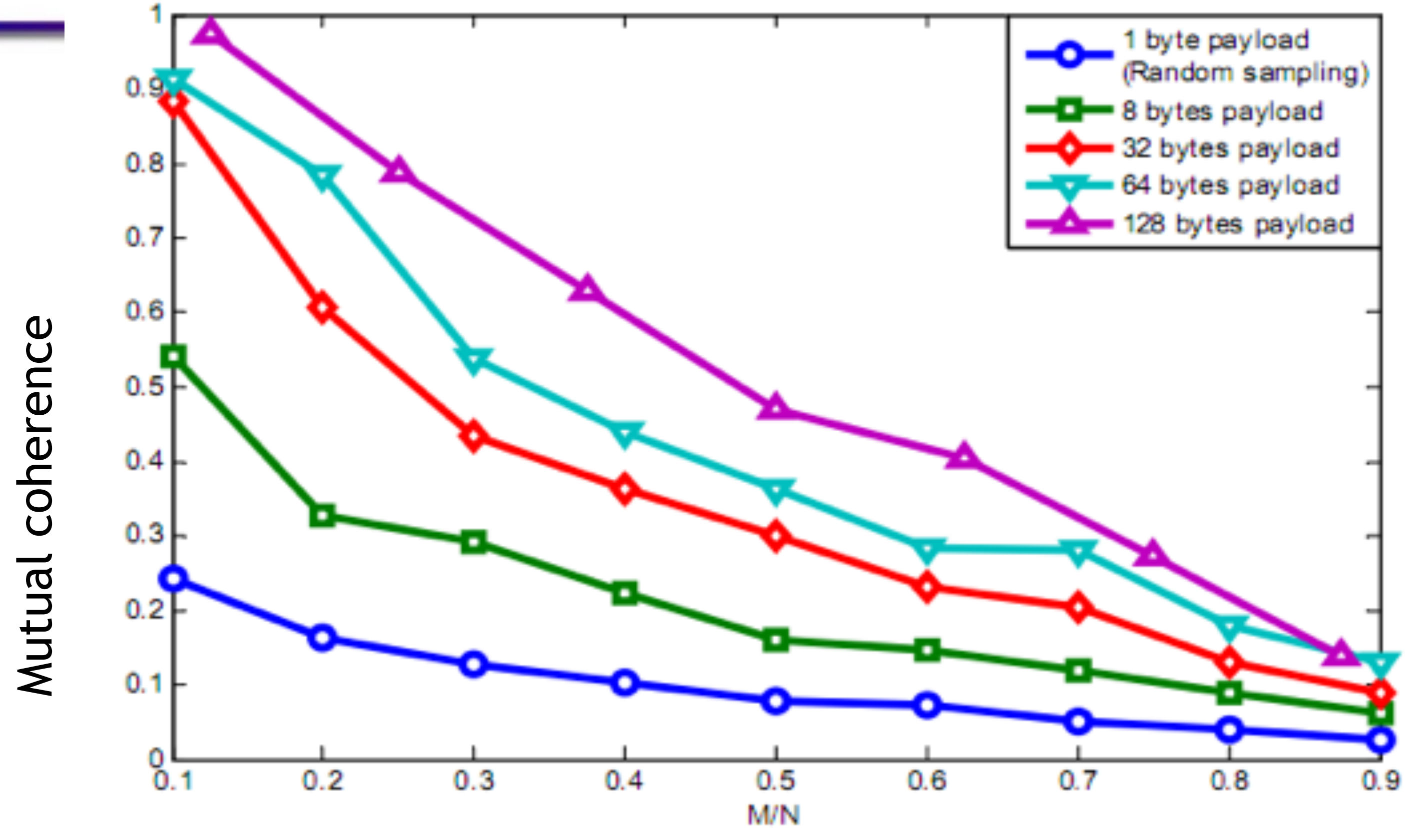
which represents the worst case coherence between any two columns (atoms) of equivalent matrix

$$A = \Phi\Psi$$

Gram matrix $G = \tilde{A}^T \tilde{A}$, where $\tilde{A}$ is column-normalized version of A

$$\mu(\mathbf{A}) = \max_{i \neq j, 1 \leq i,j \leq N} |g_{ij}|$$

Packet length effect on mutual coherence



M/N (PRR)

Packet length ↑ Mutual coherence ↑

M/N ↑ Packet length effect on mutual coherence ↓

Packet length effect on mutual coherence

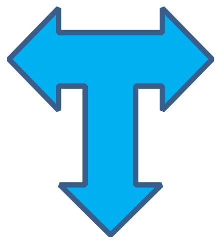
p12

Packet length ↑ Mutual coherence ↑

M/N ↑ Packet length effect on mutual coherence ↓

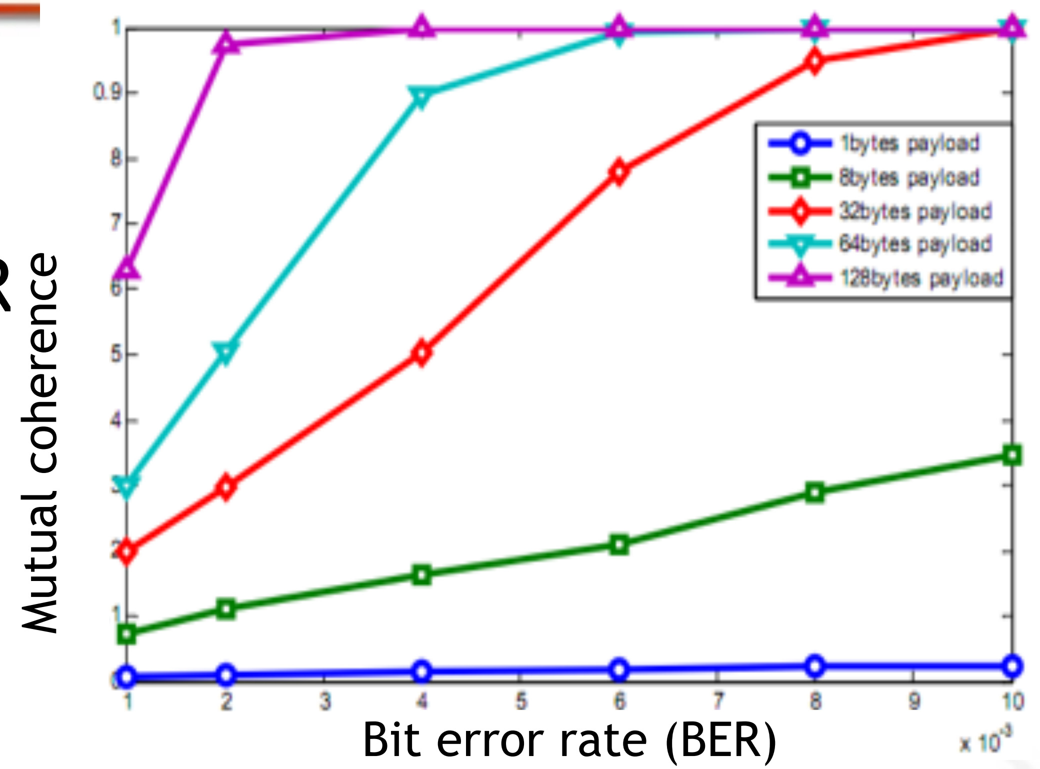
Packet length effect (BER) on mutual coherence

Packet length



BER

PRR



$$P_{prr} = (1 - P_e)^L \geq \frac{M_{threshold}}{N}$$

where

$$M_{threshold} = cK \log(N / K)$$

Packet length control

Performance improvement

Larger packet length leads to under relative good wireless situation
High transmission efficiency

?

Larger packet length leads to
Larger bursty packet loss and Larger mutual coherence

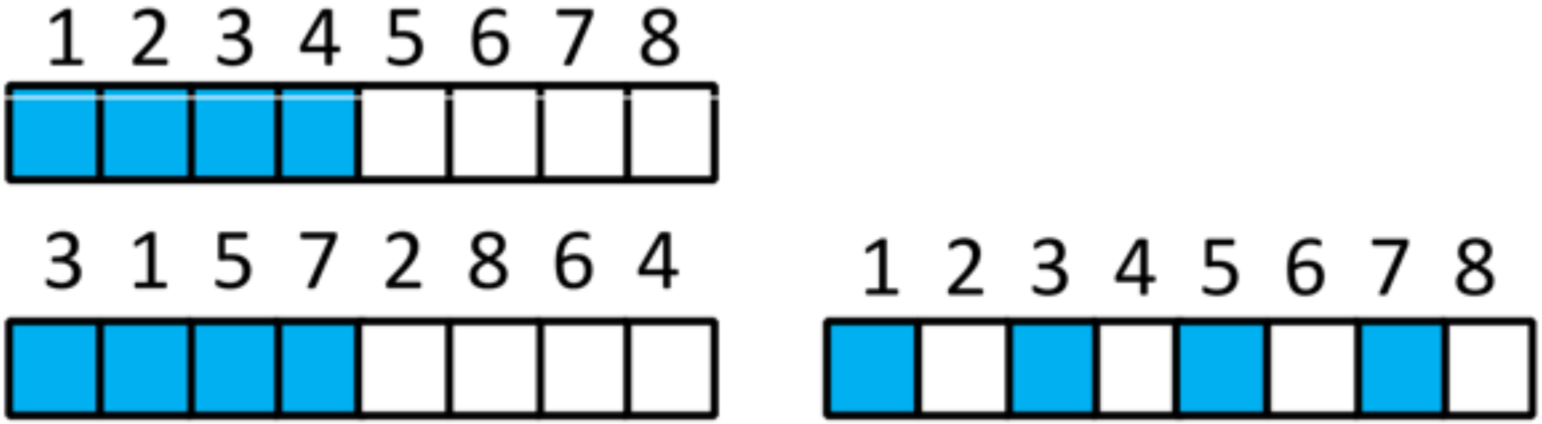
Shorter packet length leads to
random packet loss and smaller mutual coherence

How to eliminate packet length effect
to use larger length to gain efficiency

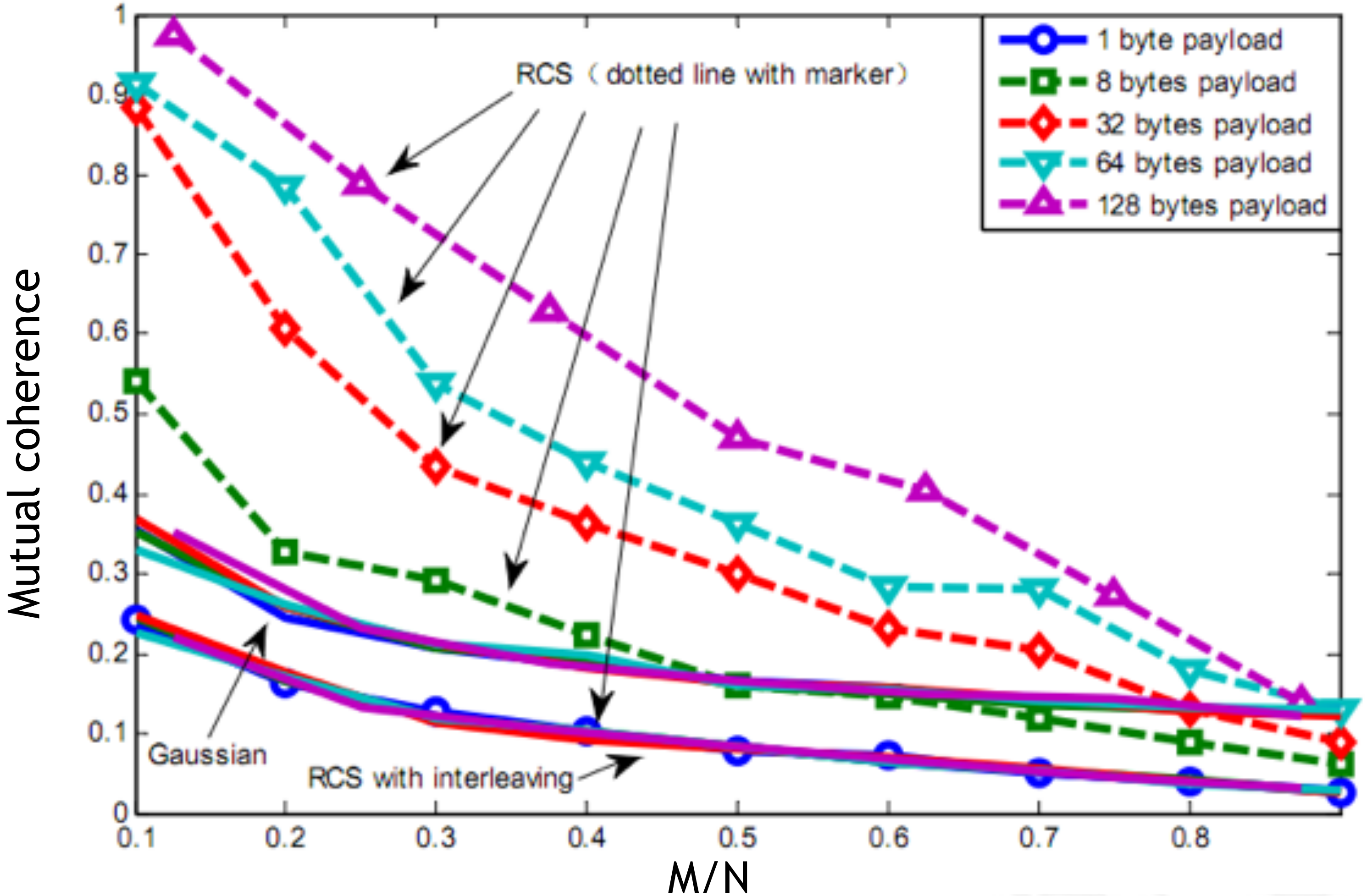
Performance improvement

Data interleaving

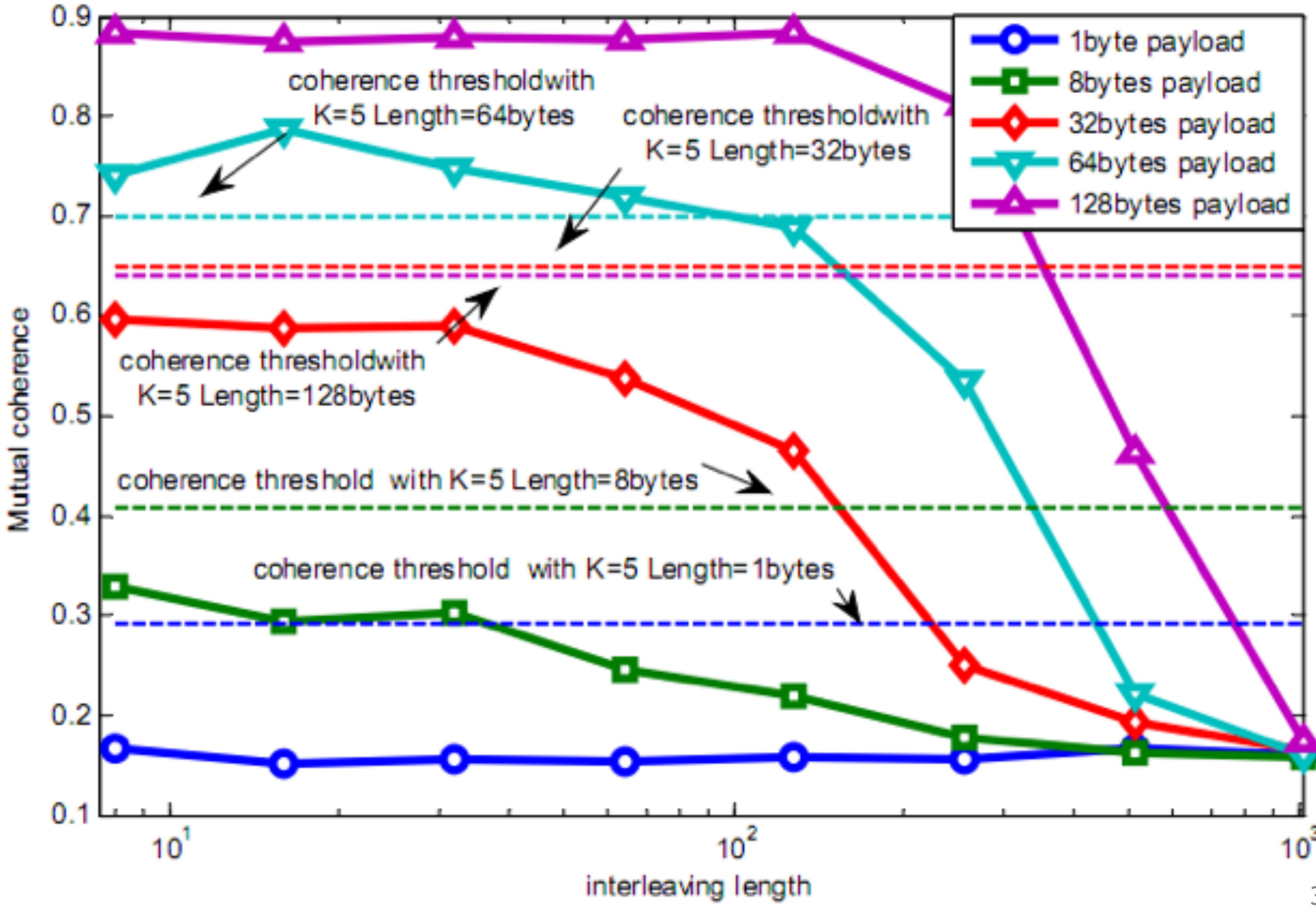
Transmits data bits in a different order than the order in which the data bits are originally transmitted



Performance improvement through data interleaving



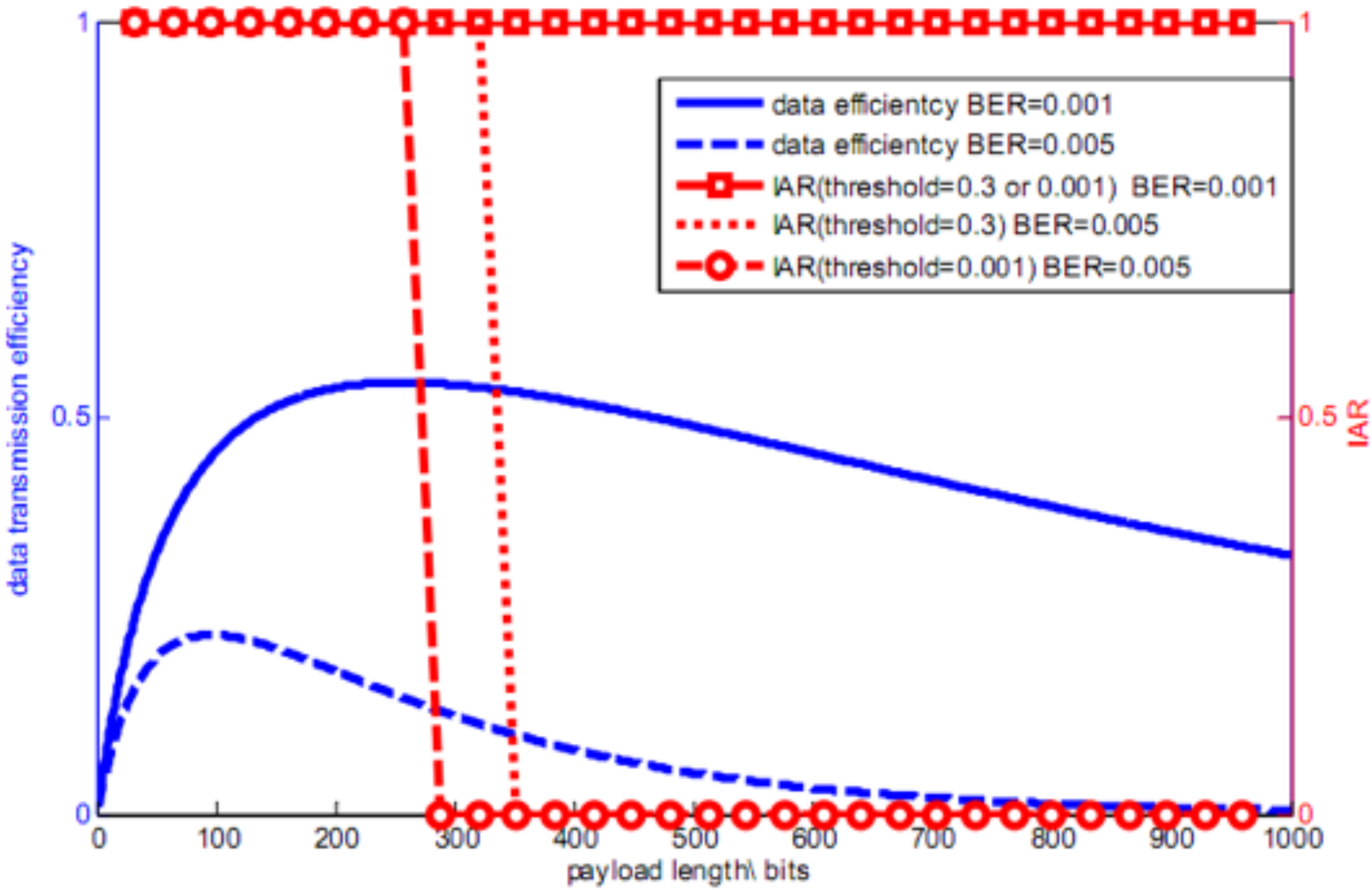
Interleaving length effect



Simulation Results

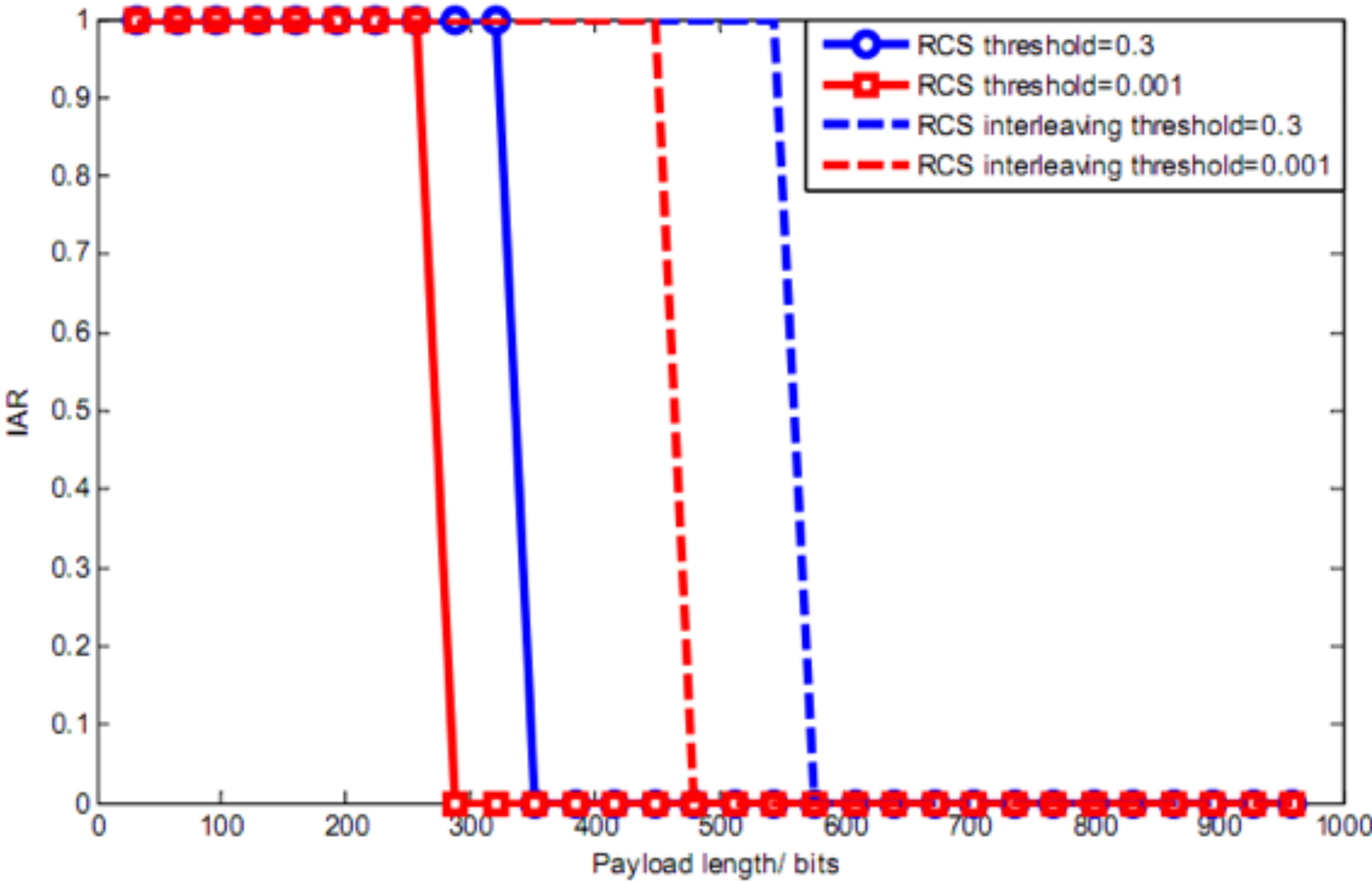
Performance comparison(data efficiency and IAR)

Varying Error threshold
Varying BER



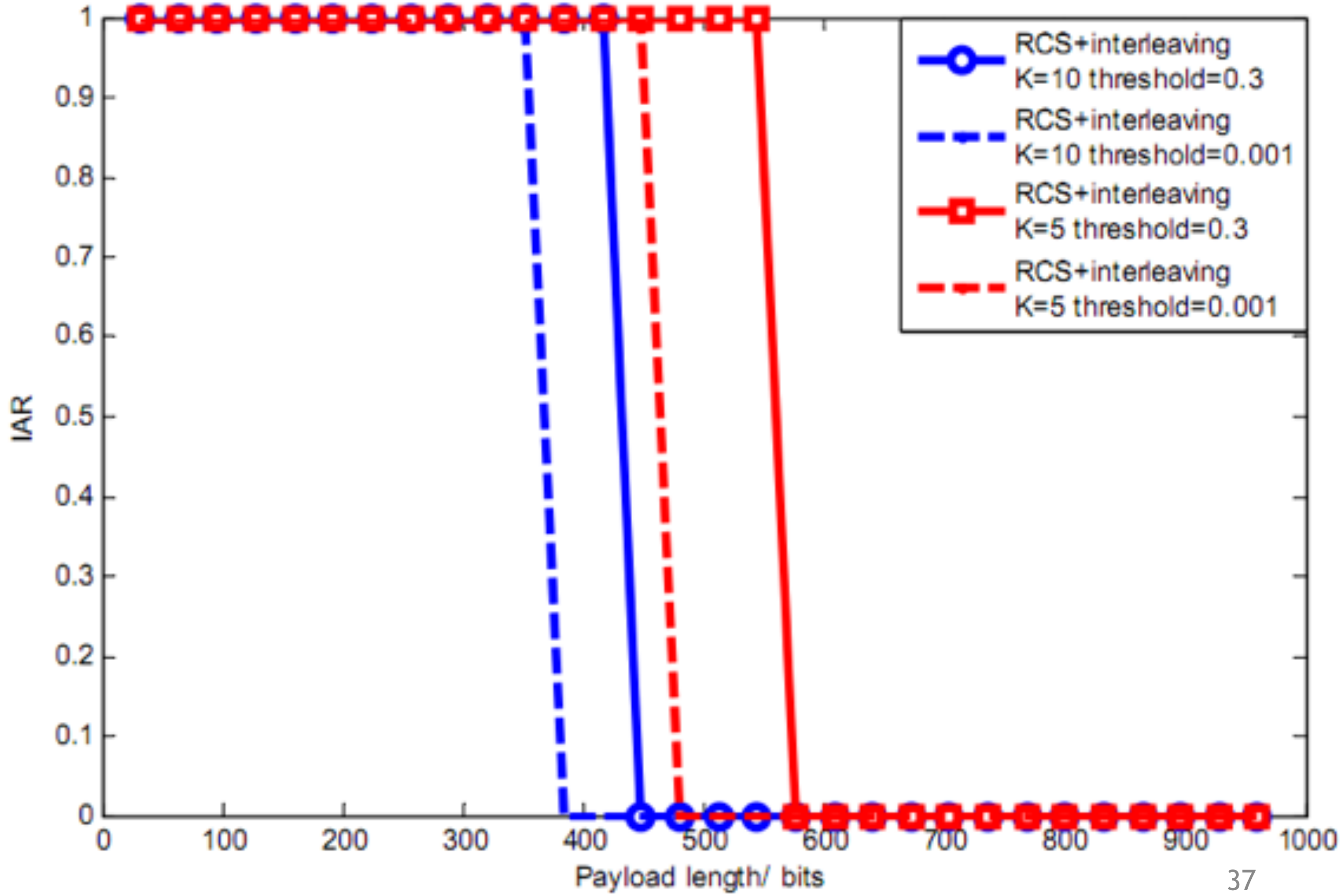
Interleaving improvement

Varying Error threshold
BER=0.005



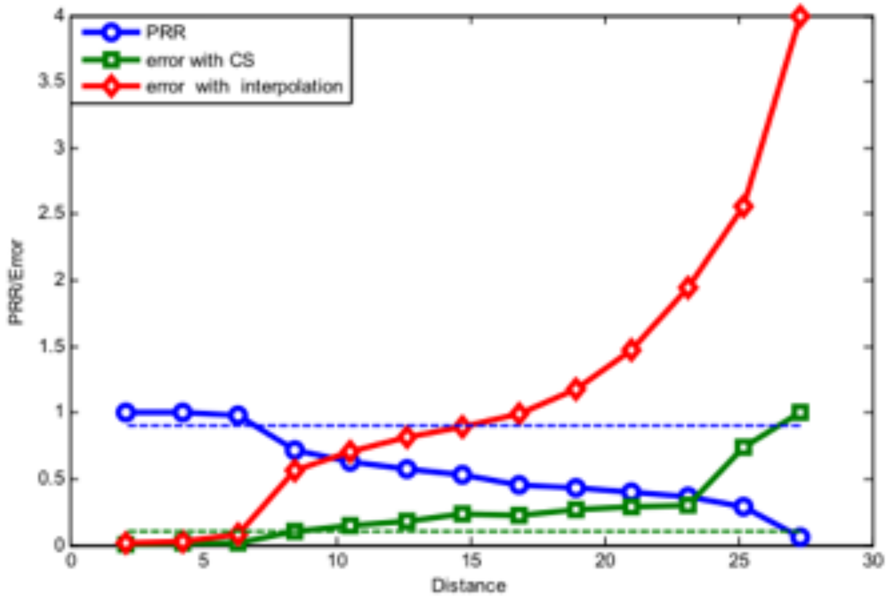
Performance comparison (varying application/sparsity)

Varying Error threshold
Varying sparsity

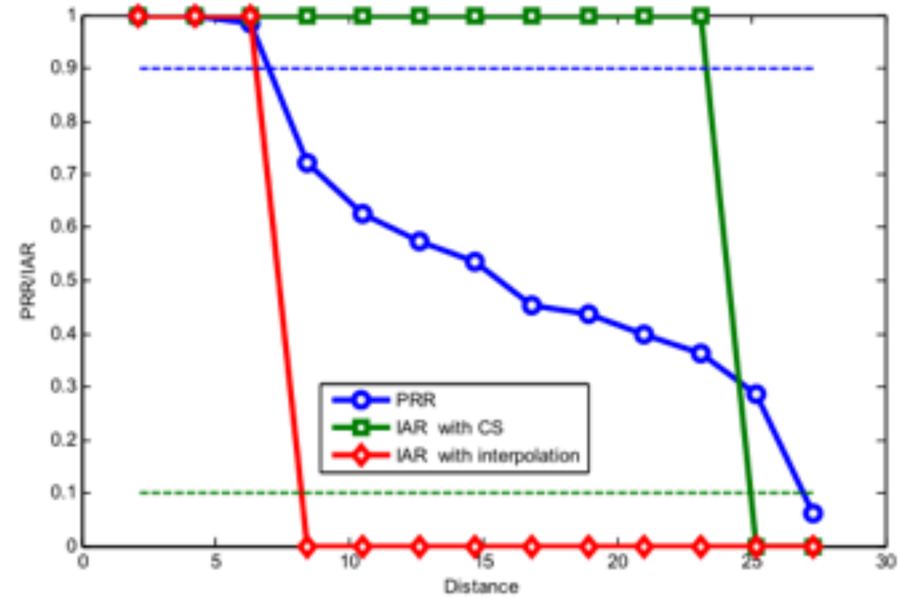


Performance Promotion

Broaden Communication range

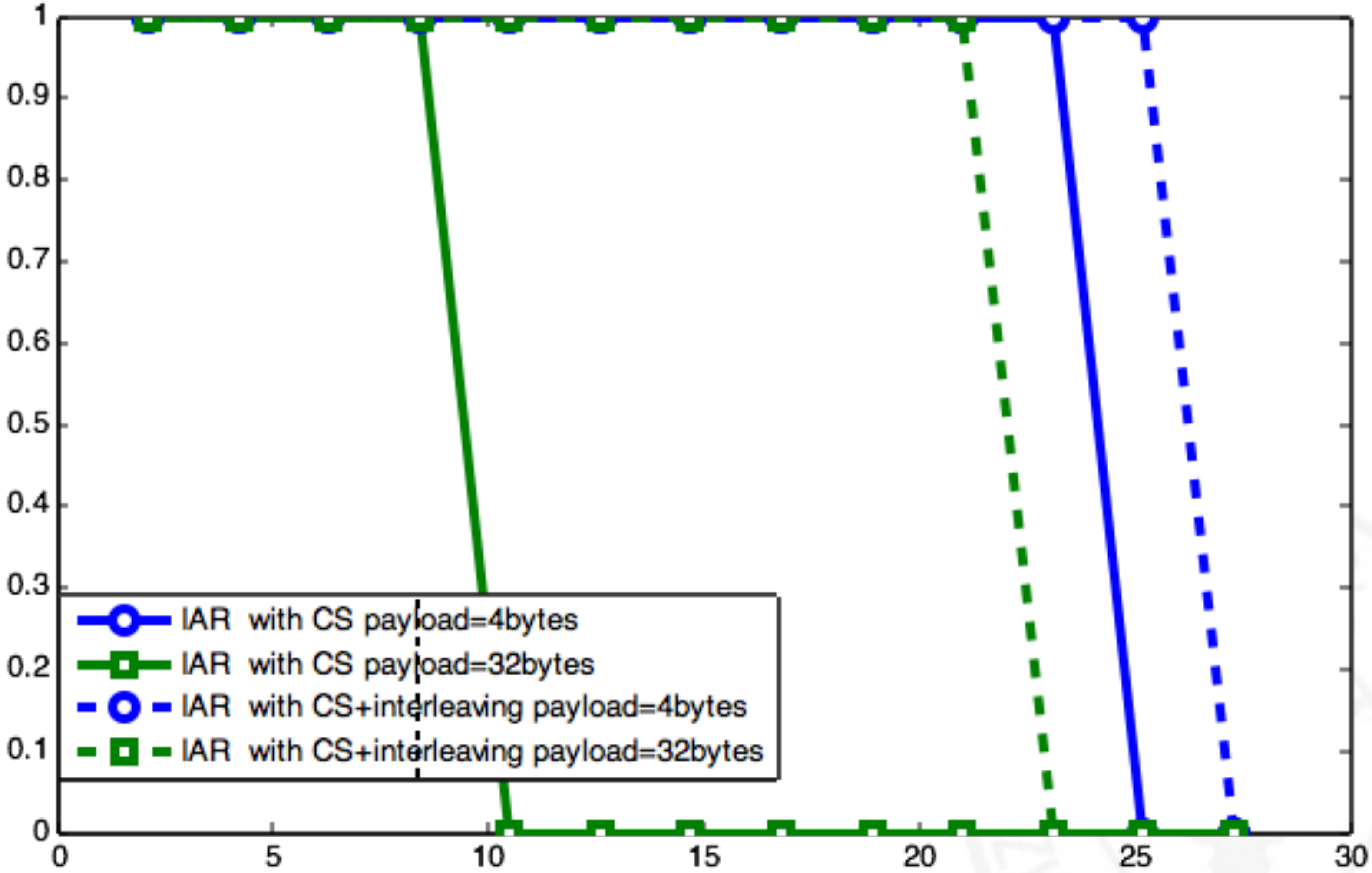


(a) PRR and Recovery error comparison with varying transmitting distance



(b) PRR and IAR comparison with varying transmitting distance

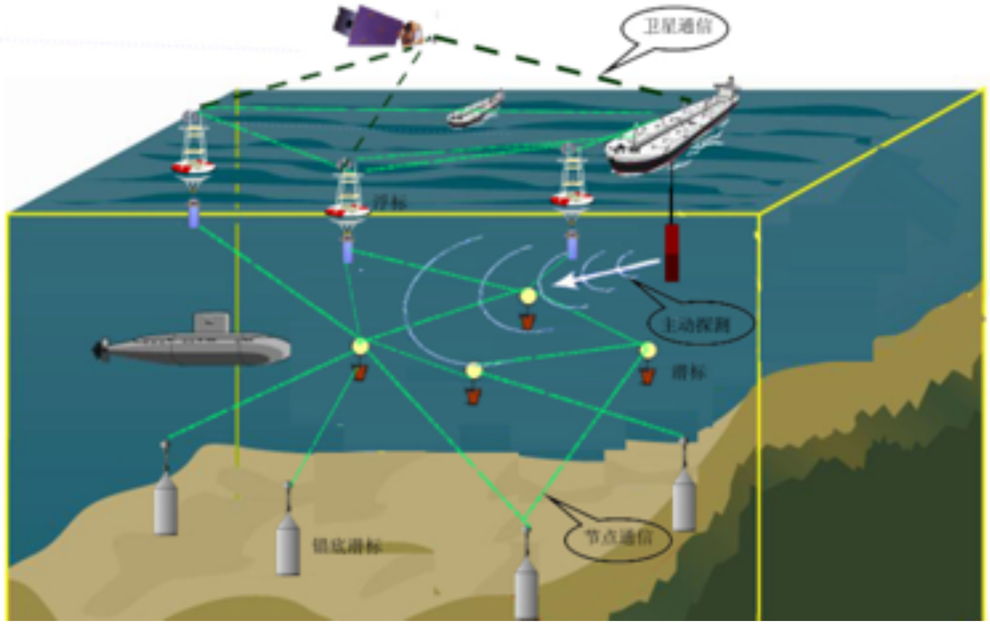
Packet length effect and interleaving improvement

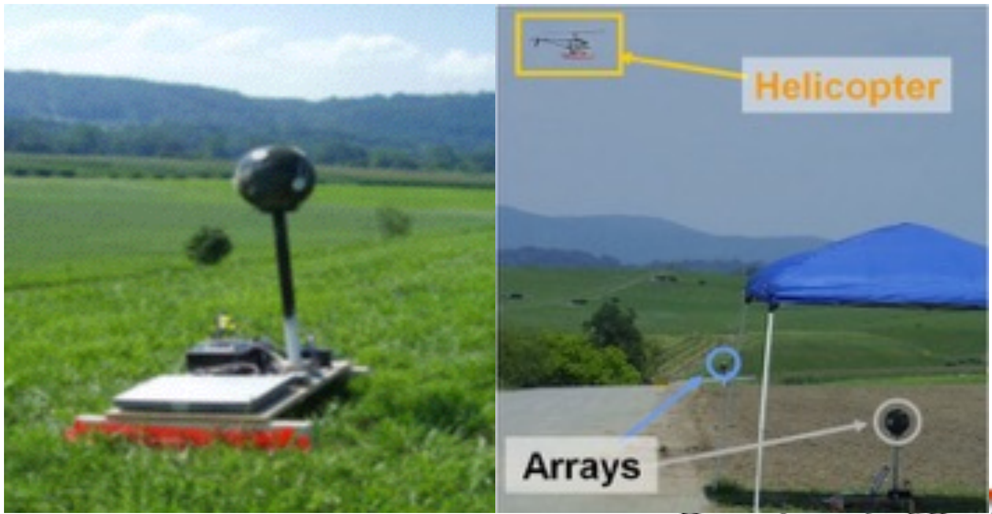


Part2

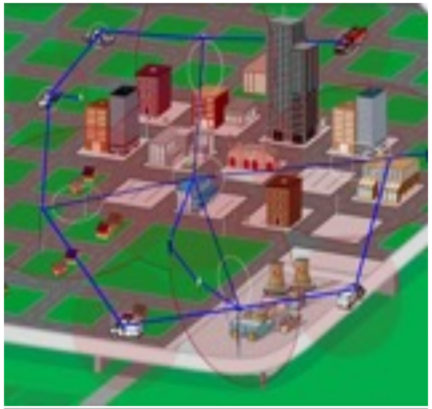
WSAN DoA Estimation from compressed Array Data via Joint Sparse Representation

Motivation - Target monitoring on sensor array





Low latitude detection



City monitoring

Gunfire positioning

Border warning

Motivation - Bottleneck for wireless sensor array

– Challenged on Wireless Platform

Data Transmission

- ➢ Wireless sensor network can support long time monitoring with no more than 1000 Hz sampling rate for IEEE 802.15.4 protocol

Power Consumption

- ➢ The development of the battery capacity is limited

Local computation capacity

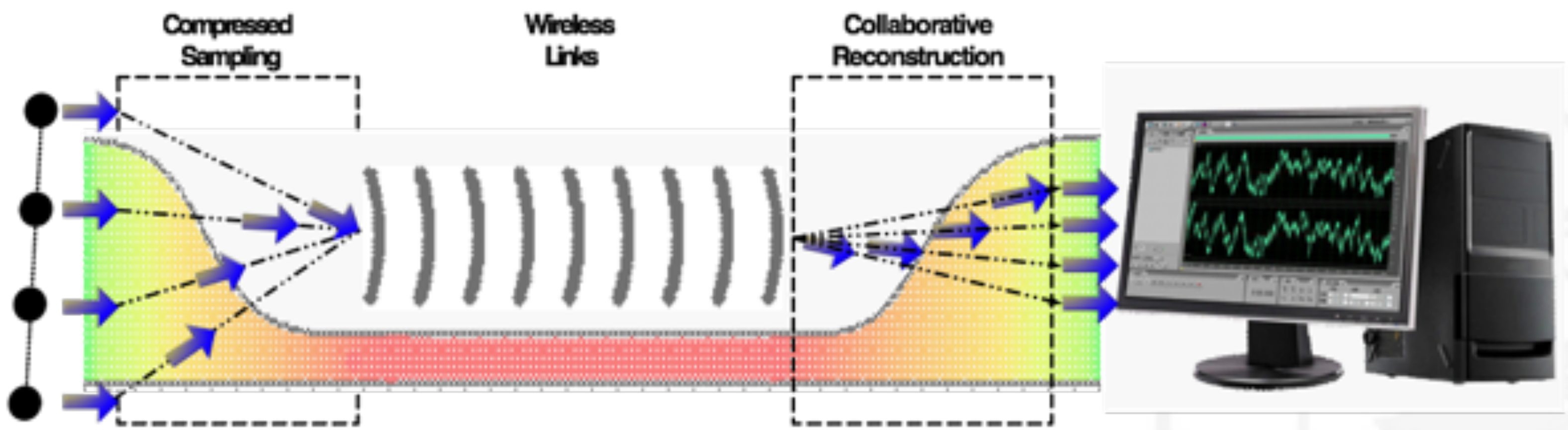
- ➢ Local computation capacity is limited under power constraint

Cost

- ➢ Implementation for large number of sensor is not affordable

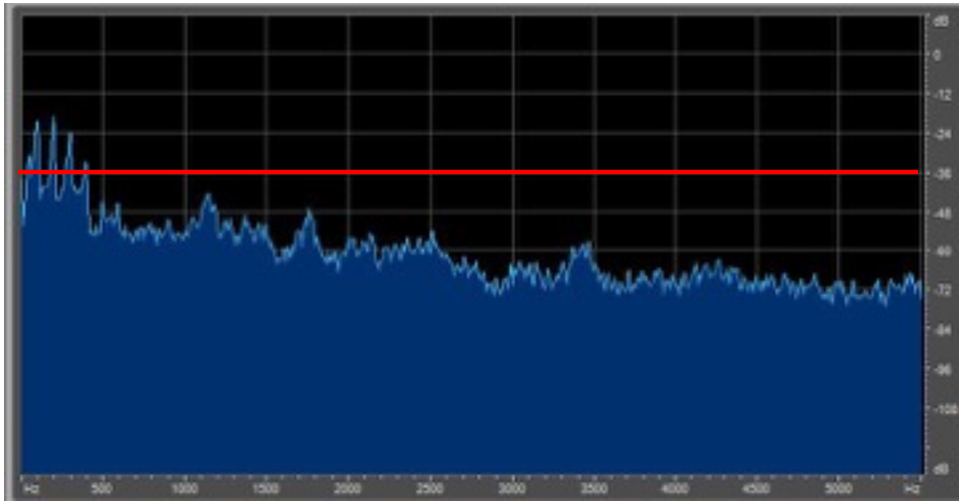
Motivation - Solution to challenges

- A Compressive Sensing based array sensor network for target monitoring
  - Compressed Sampling is introduced
  - Fusion center with strong computational capacity

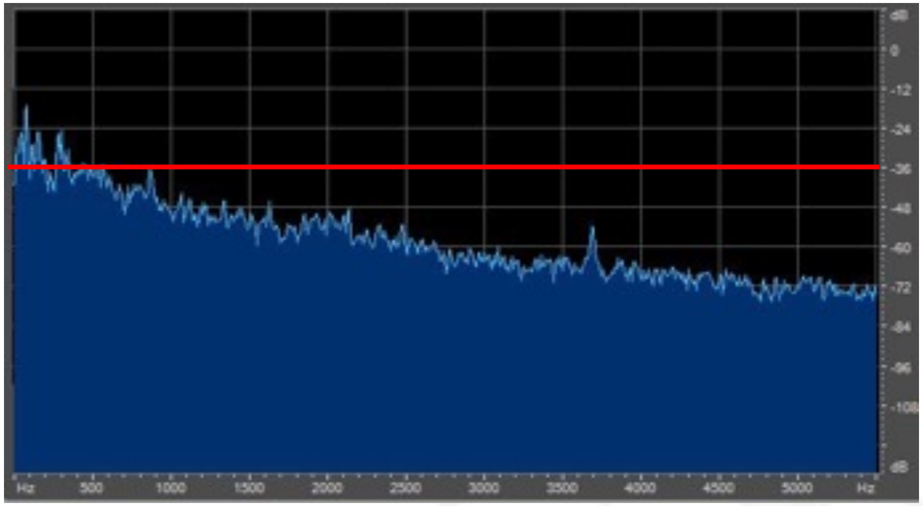


Background: Sparse representation

- Spectrum sparsity for time domain signal



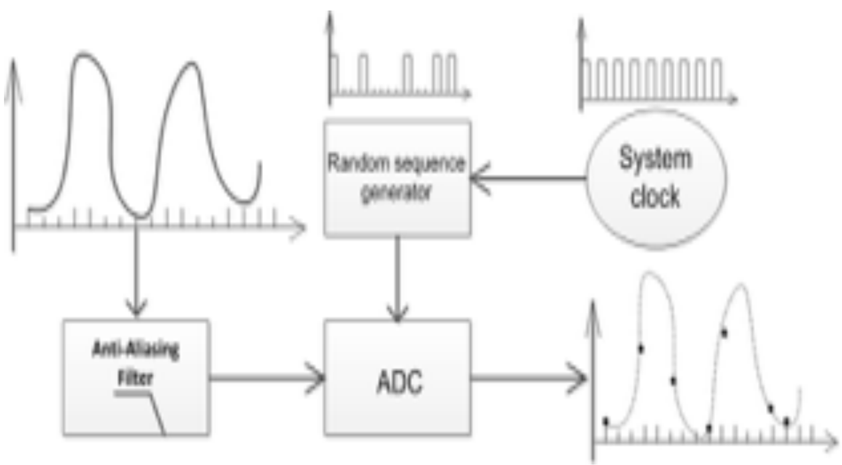
Automobile engine

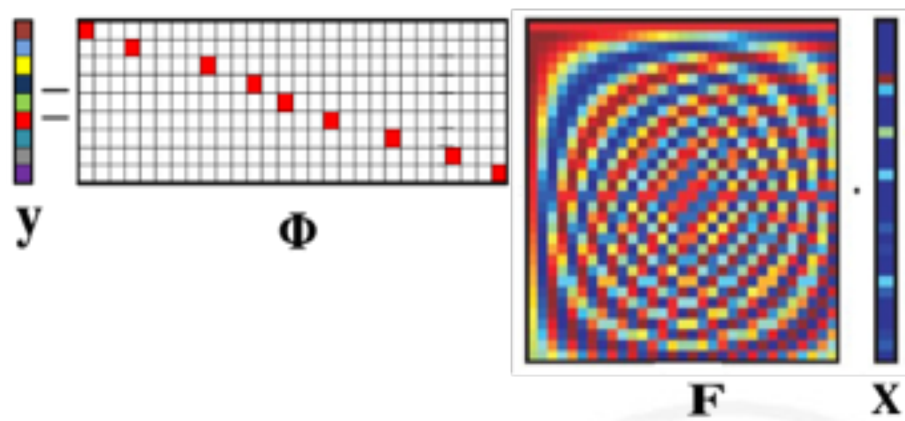


Heavy vehicle

Backgrounds: Compressive Sensing

- Efficient implementation of CS in sensor node



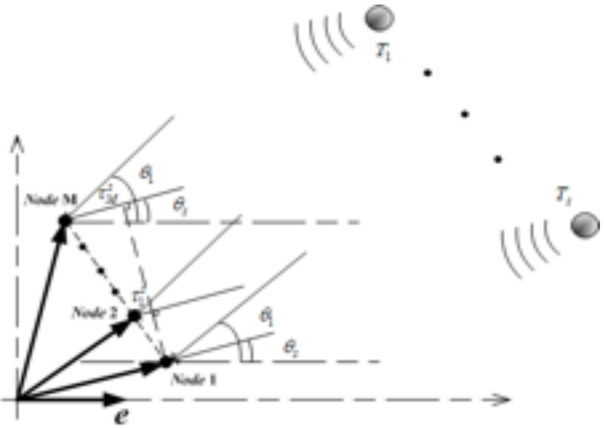


$$\mathbf{y} = \qquad + \varepsilon = \qquad + \varepsilon$$

$$\min \|\mathbf{X}\|_1 \text{ s.t. } \|\mathbf{y} - \qquad \|_2^2 \le \delta^2$$

Backgrounds: Array processing

Array Signal Model & Processing



$$x_j(n) = \sum_{q=1}^{Q} s_q(n - \tau_{q,j}) + v_j(n), n = 1, 2, ..., N$$

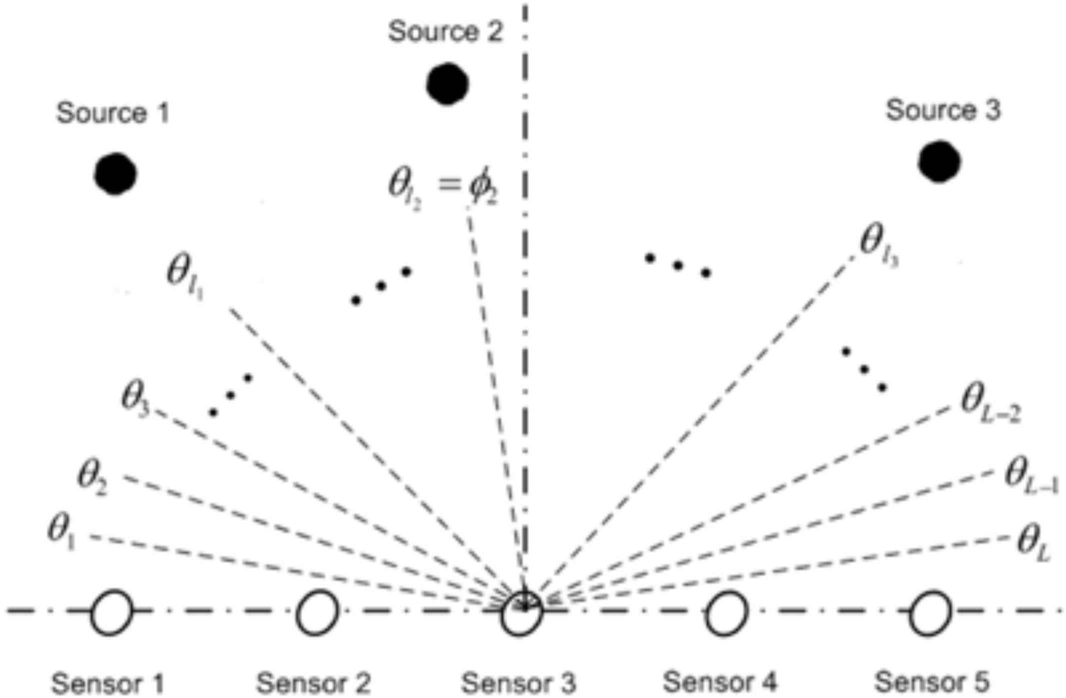
$$X_j(\omega_k, t) = \sum_{q=1}^{Q} e^{-\frac{i2\pi k\tau_{q,j}}{N}} S_q(\omega_k, t) + V_j(\omega_k, t)$$

$$X(\omega_k, t) = [X_1(\omega_k, t), X_2(\omega_k, t), \ldots, X_J(\omega_k, t)]^T$$

$$X(\omega_k, t) = \sum_{q=1}^{Q} a(\omega_k, \theta_q) S_q(\omega_k, t) + V(\omega_k, t)$$

Backgrounds: Sparse representation

- Sparse representation in angle domain

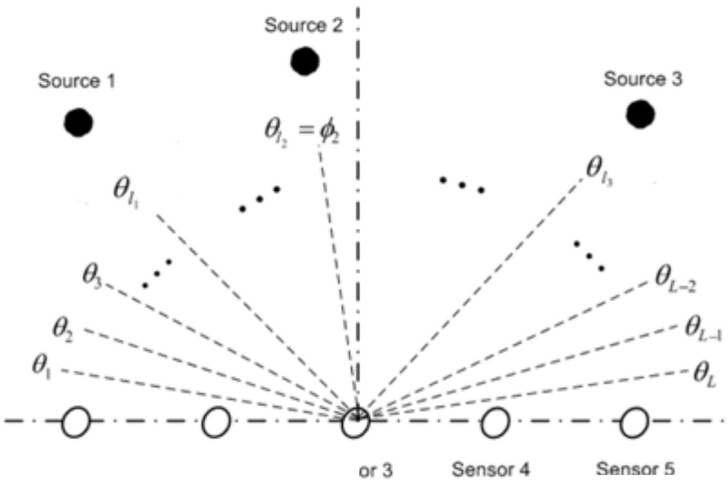


Only small number of active sources in the angle domain

Backgrounds: Array processing

Sparsity based array processing

Partition the bearing angle space (say, 0 to 180) into L bins:



$$X(\omega_k,t) = \sum_{q=1}^{Q} a(\omega_k,\theta_q) S_q(\omega_k,t) + V(\omega_k,t) \quad \Rightarrow \quad X(\omega_k,t) = \mathbf{A}(\omega_k) S(\omega_k,t) + V(\omega_k,t)$$

$$\min \quad \|S(\omega_r,t)\|_1 \quad s.t. \quad \|X(\omega_k,t) - \mathbf{A}(\omega_k) S(\omega_k,t)\|^2 \le N\delta^2$$

$$\mathbf{A}(\omega_r) = [a(\omega_r,\theta_1), a(\omega_r,\theta_2), \ldots, a(\omega_r,\theta_L)]$$

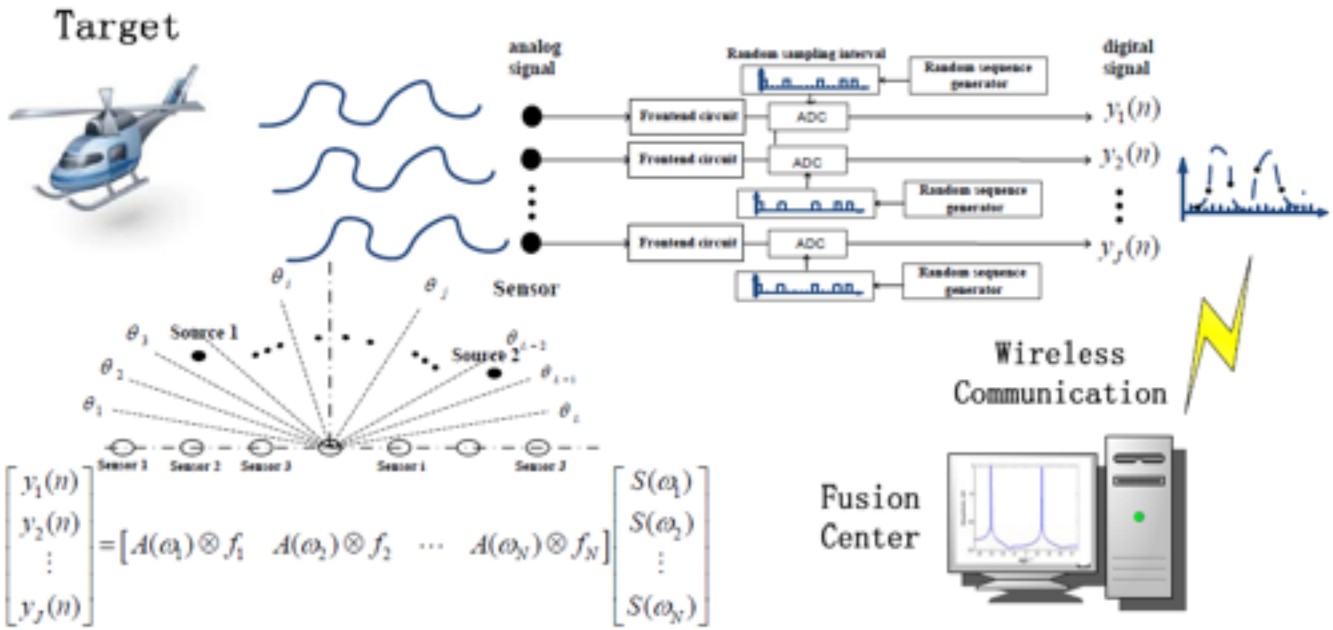
$$a(\omega_r,\theta_q) = [1, e^{-i\omega_r d \sin(\theta_q)/c}, \ldots, e^{-i\omega_r (J-1) d \sin(\theta_q)/c}]^T$$

$$S(\omega_r) = [s(\omega_r,1), s(\omega_r,2), \ldots, s(\omega_r,L)]^T$$

Framework

Combination of CS and array processing

Compressive Sensing Joint Sparse Representation based DoA



Compressed sampling /data compression → Signal recovery/data decompression → Array processing → Localization

Compressed sampling → DoA reconstruction → Localization

Problem Formulation:

Joint Compressive Sensing

Combining all J sensor measurements at the fusion center, one may write:

$$\underbrace{\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_J(t) \end{bmatrix}}_{Y\hat{(t)}} = \underbrace{\begin{bmatrix} \boldsymbol{F}_1 & & & \\ & \boldsymbol{F}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{F}_J \end{bmatrix}}_{\hat{\boldsymbol{F}}} \underbrace{\begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_J(t) \end{bmatrix}}_{\hat{X}(t)} + \underbrace{\begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_J(t) \end{bmatrix}}_{\hat{V}(t)}$$

Problem Formulation:

Joint Array Processing

Dividing the frequency band into N narrow-band, non-overlapping frequency bins, one may derive a joint sparse representation of the sensor measurements as

$$\underbrace{\begin{bmatrix} X(\omega_1, t) \\ X(\omega_2, t) \\ \vdots \\ X(\omega_N, t) \end{bmatrix}}_{\hat{X}(\omega, t)} = \underbrace{\begin{bmatrix} A(\omega_1) & & & \\ & A(\omega_2) & & \\ & & \ddots & \\ & & & A(\omega_N) \end{bmatrix}}_{\hat{\boldsymbol{A}}} \underbrace{\begin{bmatrix} S(\omega_1, t) \\ S(\omega_2, t) \\ \vdots \\ S(\omega_N, t) \end{bmatrix}}_{\hat{S}(t)}$$

Problem Formulation:

$$\underbrace{\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_J(t) \end{bmatrix}}_{\hat{Y}(t)} = \underbrace{\begin{bmatrix} \boldsymbol{F}_1 & & & \\ & \boldsymbol{F}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{F}_J \end{bmatrix}}_{\hat{\boldsymbol{F}}} \underbrace{\begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_J(t) \end{bmatrix}}_{\hat{X}(t)} + \underbrace{\begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_J(t) \end{bmatrix}}_{\hat{V}(t)} \qquad \underbrace{\begin{bmatrix} X(\omega_1, t) \\ X(\omega_2, t) \\ \vdots \\ X(\omega_N, t) \end{bmatrix}}_{\hat{X}(\omega,t)} = \underbrace{\begin{bmatrix} A(\omega_1) & & & \\ & A(\omega_2) & & \\ & & \ddots & \\ & & & A(\omega_N) \end{bmatrix}}_{\hat{\boldsymbol{A}}} \underbrace{\begin{bmatrix} S(\omega_1, t) \\ S(\omega_2, t) \\ \vdots \\ S(\omega_N, t) \end{bmatrix}}_{\hat{S}(t)}$$

$$\hat{X}(t) \xrightarrow{\;?\;} \hat{X}(\omega,t) \qquad \text{unvec}_{J,N}(\hat{X}(\omega,t)) = \text{unvec}_{N,J}(\hat{X}(t))^T$$

$$\hat{X}(t) = \boldsymbol{H}\hat{X}(\omega,t)$$

$$\hat{Y}(t) = \hat{\boldsymbol{F}}\hat{X}(t) + \hat{V}(t) = \hat{\boldsymbol{F}}\boldsymbol{H}\hat{X}(\omega,t) + \hat{V}(t) = \hat{\boldsymbol{F}}\boldsymbol{H}\hat{\boldsymbol{A}}\hat{S}(t) + \hat{V}(t) = \boldsymbol{\Theta}\hat{S}(t) + \hat{V}(t)$$

$$\min \quad \|\hat{S}(t)\|_{2,1} \quad s.t. \quad \|\hat{Y}(t) - \boldsymbol{\Theta}\hat{S}(t)\|_2 \leq J\delta^2 \qquad \|\hat{S}(t)\|_{2,1} = \sum_{l=1}^{L}\sum_{t=1}^{T}\sum_{n=1}^{N} S_l(\omega_n, t)^2$$

Algorithm 1 Two step framework of joint reconstruction

Input:

The set of joint random samples in T snapshots, $\hat{Y}(t), t \in \{1, 2, \cdots, T\}$;

The measurement matrix for each sensor, $\mathbf{\Phi}_j, j \in 1, 2, \cdots, J$;

Output:

DoA indicative vector, Sp;

1: Estimate the noise level by random sampling in resource free scenario, $\delta^2 = \sum_{t=1}^{T} norm(\hat{Y}(t))^2/T$;

2: Estimate the support of source signal, $Supp(\bar{\boldsymbol{X}}), \bar{\boldsymbol{X}} = [X_1, X_2, \cdots, X_J]$ by solving:

$$\min \quad \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{j=1}^{J} X_j(n,t)^2 \quad s.t. \; ||\hat{Y}(t) - \hat{\boldsymbol{F}}\hat{X}(t)||_2^2 \leq TJ\delta^2, t = 1, 2, \cdots, T$$

3: Construct the Pruned joint reconstruction matrix, $\hat{\mathbf{\Theta}}$;

$$\hat{\mathbf{\Theta}} = [\hat{\mathbf{\Theta}}[r_1] \quad \hat{\mathbf{\Theta}}[r_2] \cdots \hat{\mathbf{\Theta}}[r_i]], r_i \in Supp(\bar{\boldsymbol{X}})$$

4: Solve the pruned reconstruction problem;

$$\min \quad \sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{r \in Supp(\bar{\boldsymbol{X}})} s_l(\omega_r, t)^2 \quad s.t. \; ||\hat{Y}(t) - \hat{\mathbf{\Theta}}\hat{S}(t)||_2^2 \leq TJ\delta^2, t = 1, 2, \cdots, T;$$

5: Return Sp; $Sp(l) = \sum_{t=1}^{T} \sum_{r \in Supp(\bar{\boldsymbol{X}})} s_l(\omega_r, t)^2$

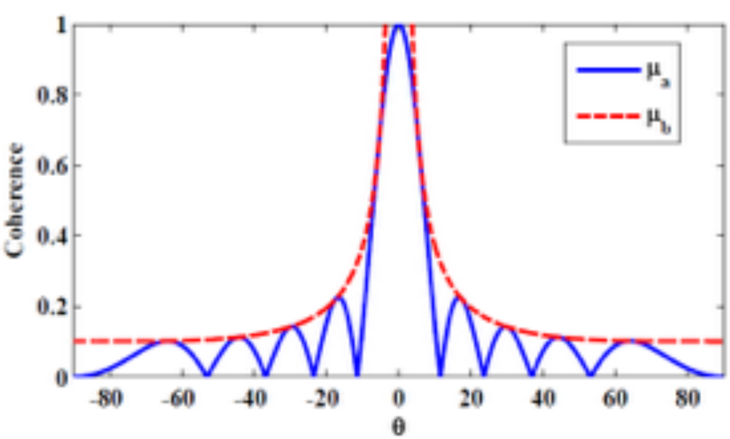
Reconstruction Analysis

Mutual coherence analysis for array manifold matrix:

Theorem 1. *Assume J sensor nodes form a uniformly spaced linear array with distance between successive sensor nodes equal to d. Also, let λ be the wave length of the acoustic signal. If $d \leq \lambda/2$ and the bearing angle difference between two sources $\Delta\theta \geq 2\,|\arcsin(\lambda/2Jd\cos(\theta_a))\,|$, then the coherence of the array manifold satisfies:*

$$\mu_a \leq \frac{1}{J}\left|\frac{2}{1-e^{(-i2\pi/J)}}\right|, \tag{23}$$

here we define θ_a is the average angle value of two sources.



$$\begin{aligned}\mu_a &= \max_{l_1\neq l_2}\frac{1}{J}\left|\sum_{j=0}^{J-1}\exp(-ij\omega dp(\theta)/c)\right| \\ &= \frac{1}{J}\left|\frac{1-\exp(-iJ\omega dp(\theta)/c)}{1-\exp(-i\omega dp(\theta)/c)}\right| \leq \frac{1}{J}\left|\frac{2}{1-\exp(-i\omega dp(\theta)/c)}\right| \equiv \mu_b\end{aligned}$$

$$\Delta\theta \geq 2\,|\arcsin(\lambda/Jd\cos(\theta_a))\,|$$

Angle partition number:

$$L = \pi/(\lfloor\arcsin(\lambda/Jd)\rfloor)$$

CRB analysis

$$CRB(\lambda_i) = \boldsymbol{F}^{-1}[\Lambda]_{i,i},$$

$$\boldsymbol{F} = 2Re[\boldsymbol{H}^H R_\sigma^{-1} \boldsymbol{H}] = (2/\delta^2)Re[\boldsymbol{H}^H \boldsymbol{H}]$$

$$Y = G(\Lambda) = \sum_{r=1}^{R} \{[\sum_{q=1}^{Q} a(\omega_r, \theta_q) S(\omega_r, q)] \otimes F(*, f_r)\}$$

$$\boldsymbol{H} = [\frac{\partial G}{\partial \theta^T}, \frac{\partial G}{\partial S^T}, \frac{\partial G}{\partial f_r^T}]$$

$$\frac{\partial G}{\partial \theta} = \sum_{r=1}^{R} \{[S(\omega_{f_r}) u(\omega_{f_r}, \theta) \odot a(\omega_{f_r}, \theta)] \otimes F(*, r)\}, u(\omega_{f_r}) = \partial a(\omega_{f_r}, \theta)/\partial \theta$$

$$\frac{\partial G}{\partial S^T} = [\frac{\partial G}{\partial S(\omega_{f_1})}, \frac{\partial G}{\partial S(\omega_{f_2})}, \ldots, \frac{\partial G}{\partial S(\omega_{f_R})}], \frac{\partial G}{\partial S(\omega_{f_r})} = a(\omega_r, \theta) \otimes F(*, r)$$

$$\frac{\partial G}{\partial f_r^T} = [\frac{\partial G}{\partial f_1}, \frac{\partial G}{\partial f_2}, \ldots, \frac{\partial G}{\partial f_R}], \frac{\partial G}{\partial f_r} = S(\omega_r) a(\omega_r) \otimes (w(\omega_r) \odot F(*, r)).$$

CRB analysis

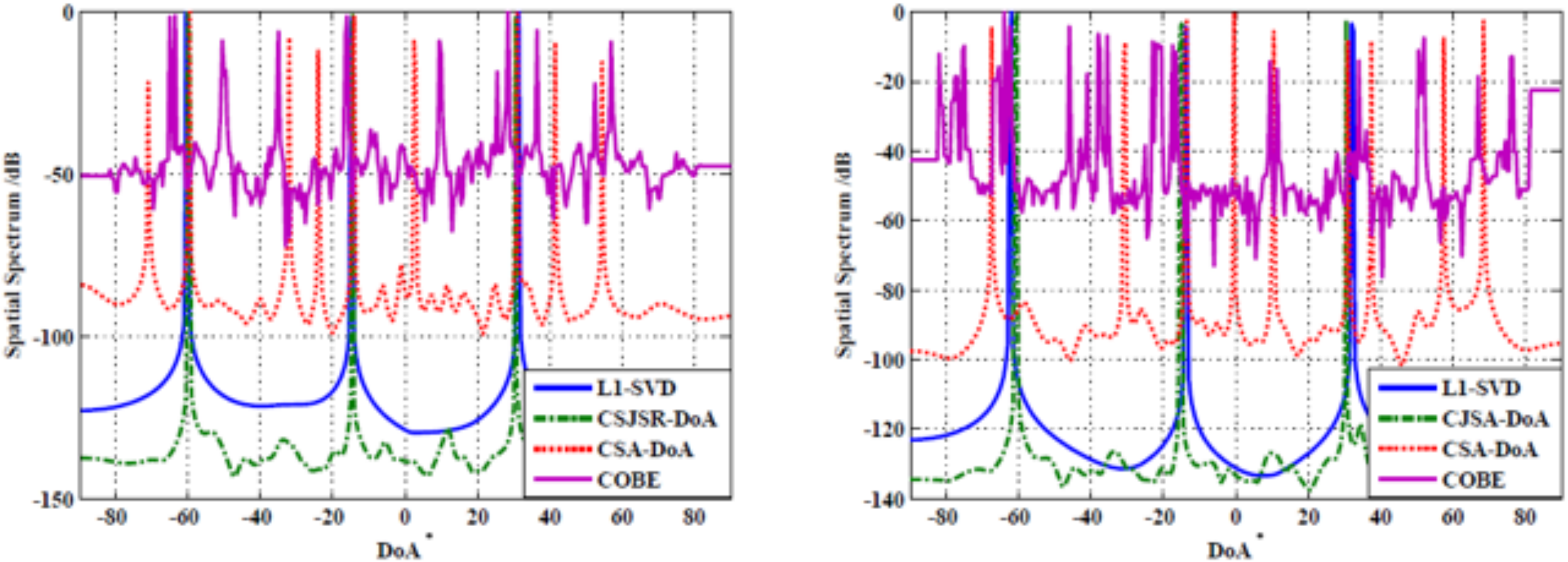
CRB of CSJSR:

$$CRB(\theta) = \frac{\delta^2}{2}\left(\sum_{r=1}^{R} S_1^2(\omega_r)(\sigma M\omega_r^2 - \frac{\rho^2\varrho^2}{\varsigma} - \frac{\rho^2(M-\varrho^2/\varsigma)}{MJ-\varrho/\varsigma})\right)^{-1}$$

CRB of traditional wideband array processing:

$$CRB_t(\theta) = \frac{\delta^2}{2N}\mathrm{Re}\left(\sum_{r=1}^{R} S^2(\omega_r)\omega_r^2\sigma - \sum_{r=1}^{R}\frac{\rho^2 S^2(\omega_r)}{J}\right)^{-1}$$

Simulation & Experiment

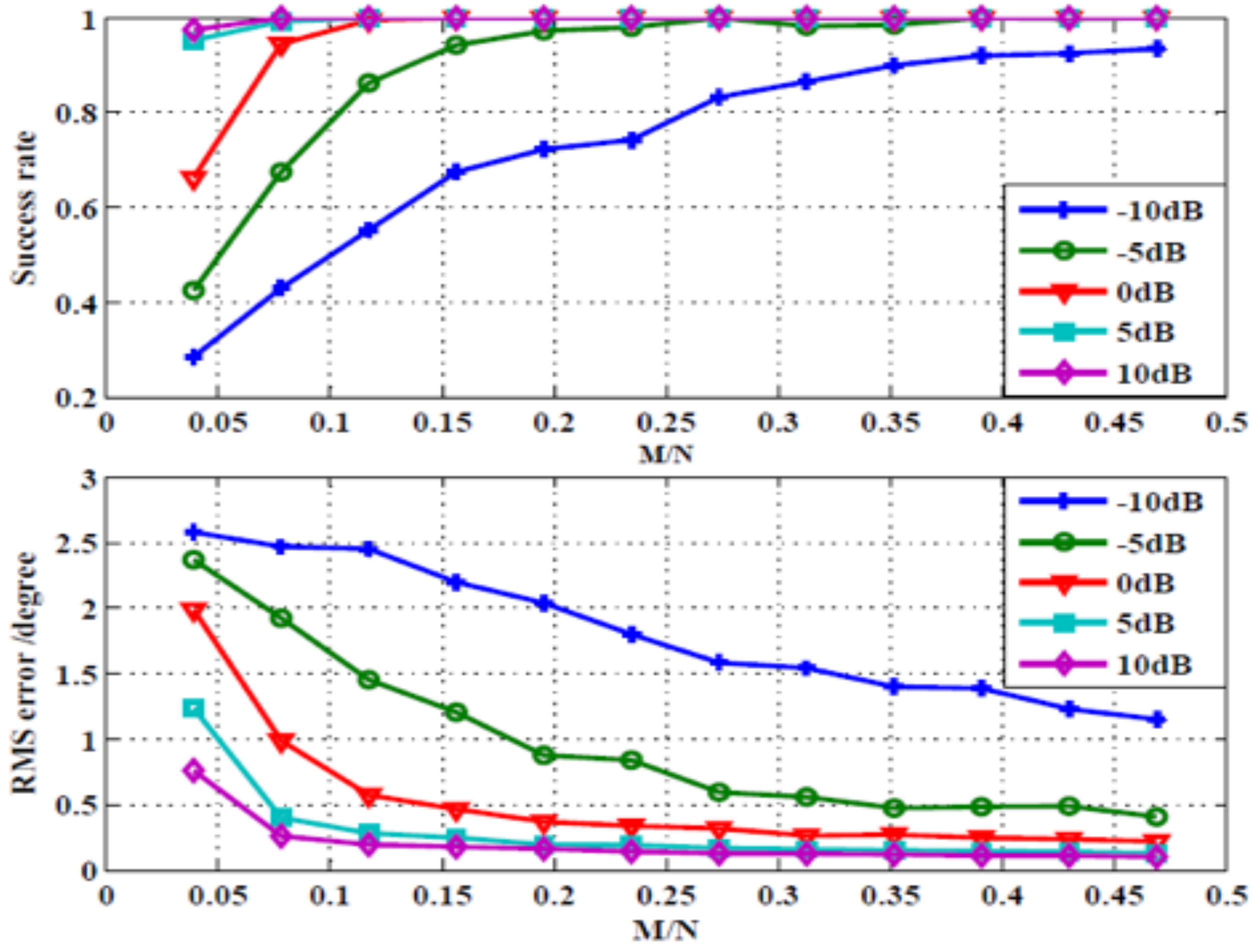


(a) SNR 10 dB (b) SNR 0 dB

L1-SVD: broadband version of L1-SVD DoA estimator
COBE : Compressive Bearing Estimation with Reference Sensor
CSJSR-DoA: Compressive Sensing based Direct DoA estimation
CSA-DoA: Compressive Sensing array DoA estimation

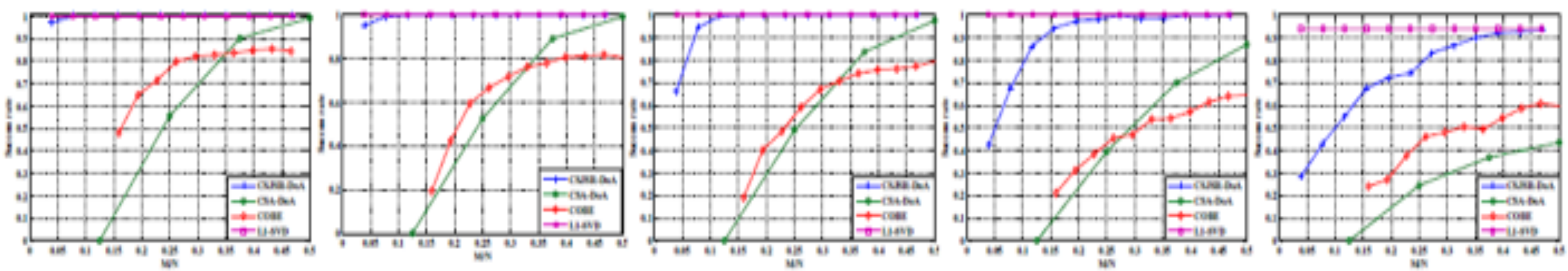
Simulation & Experiment

DoA reconstruction under different data reduction level

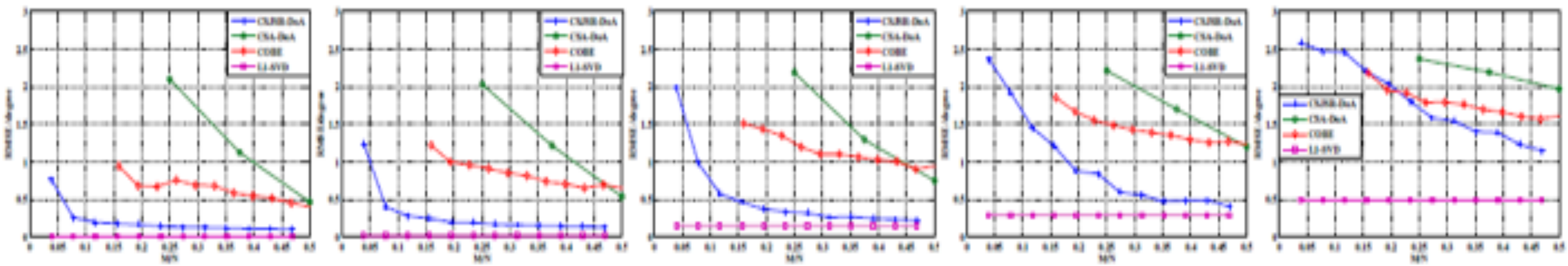


Simulation & Experiment

DoA comparison under same data volume:



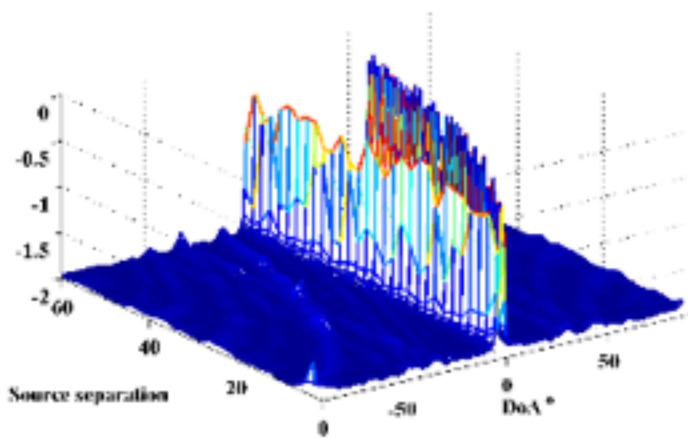
(a) Success rate 10db (b) Success rate 5db (c) Success rate 0db (d) Success rate -5db (e) Success rate -10db



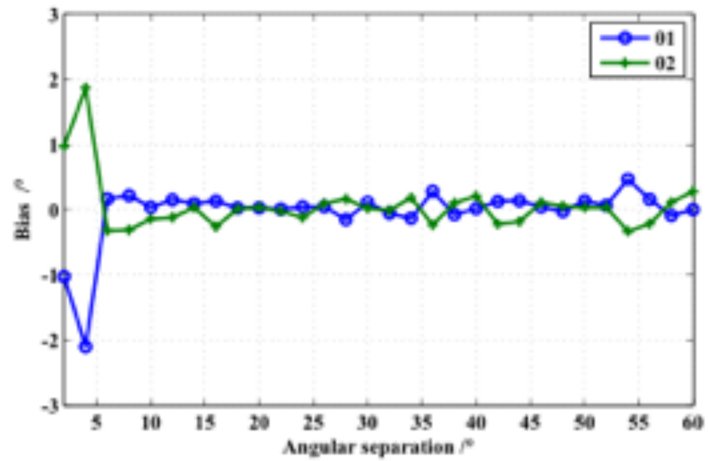
(f) RMSE 10db (g) RMSE 5db (h) RMSE 0db (i) RMSE -5db (j) RMSE -10db

Simulation & Experiment

Angular separation comparison



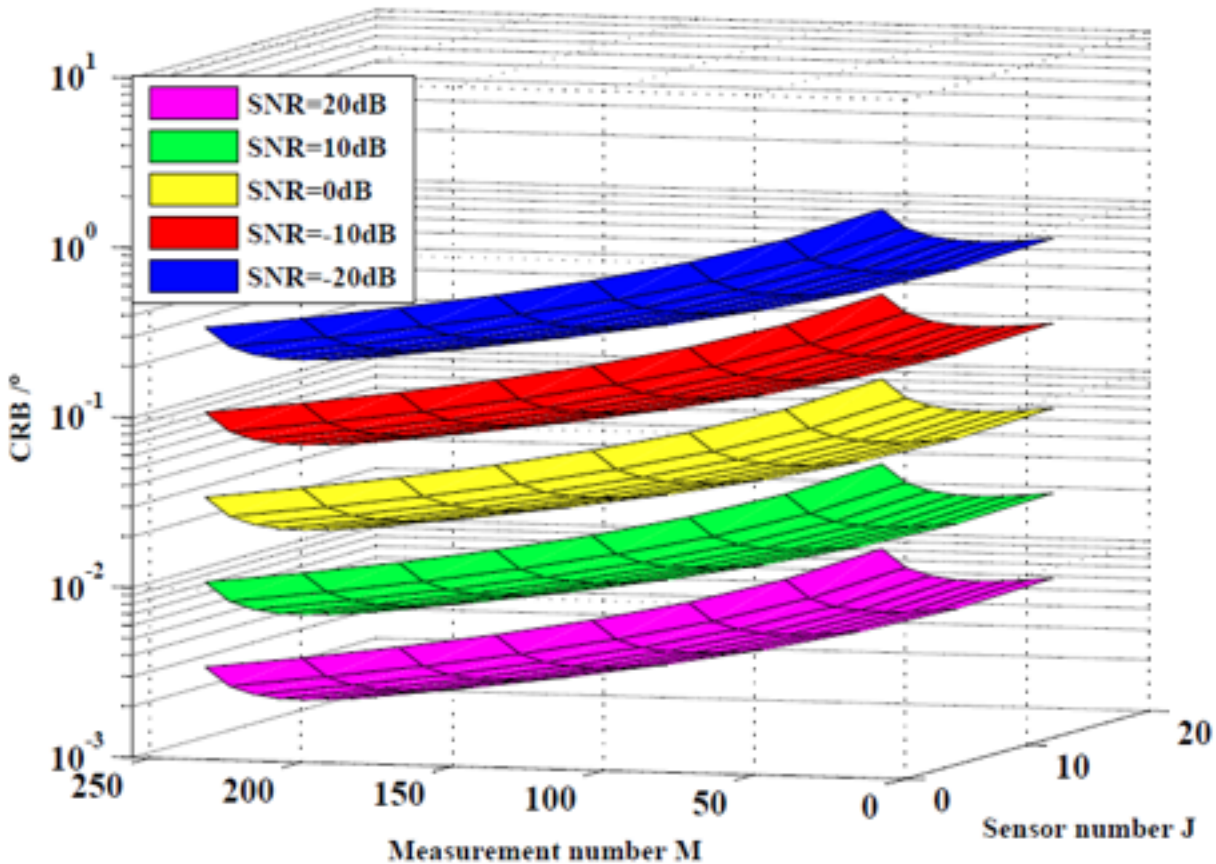
(a) Spatial spectrum of different angular distance



(b) Biases of different angular distance

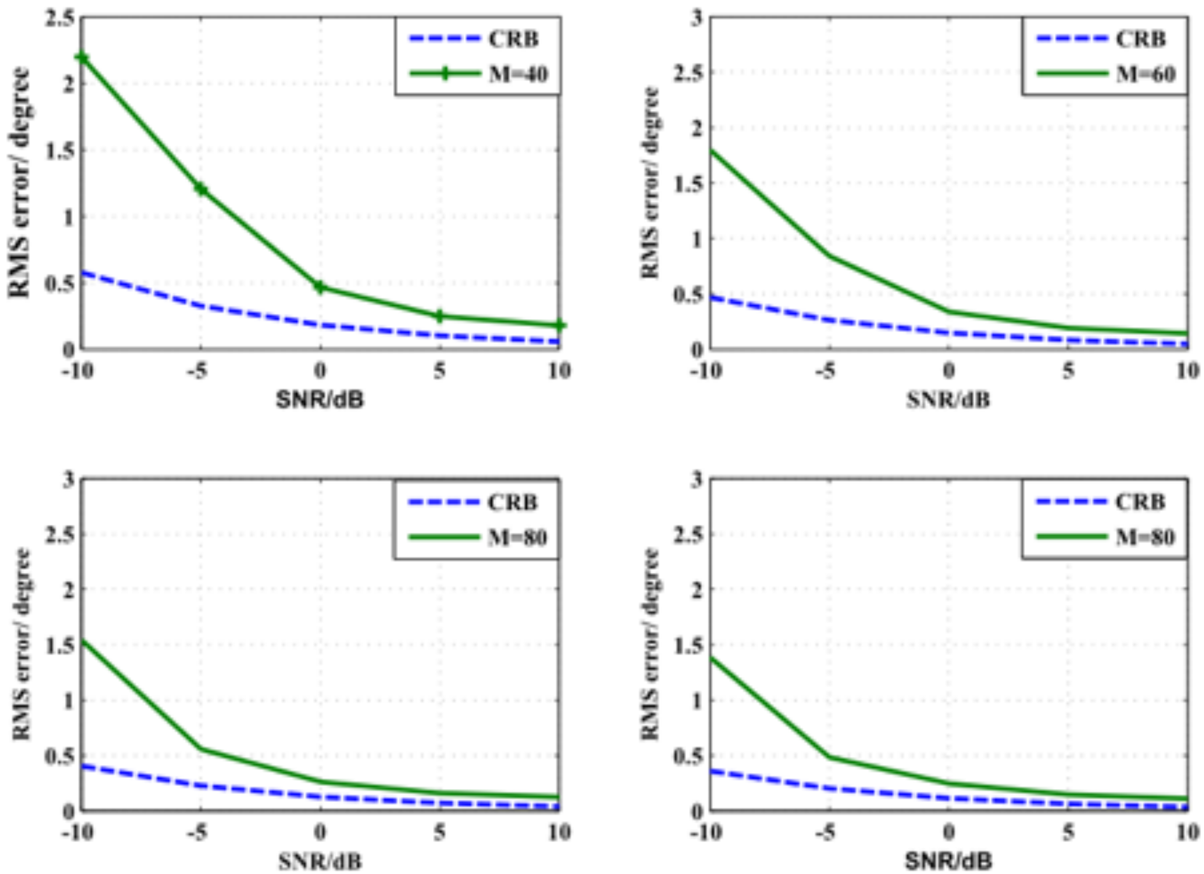
Simulation & Experiment

- CRB analysis under different J,M and SNR



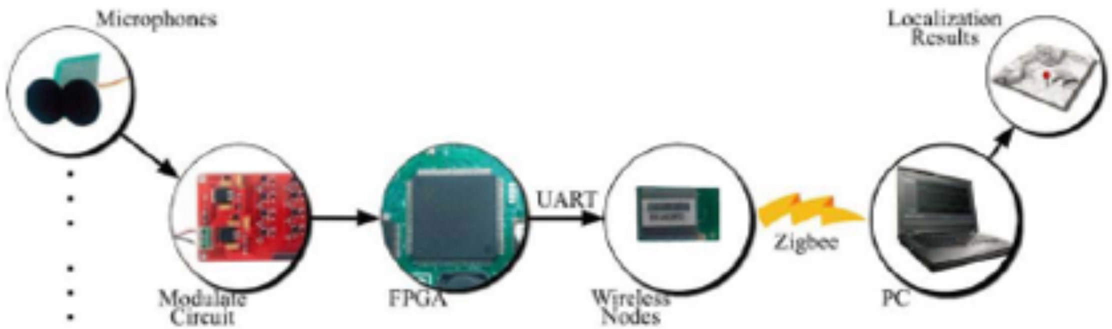
Simulation & Experiment

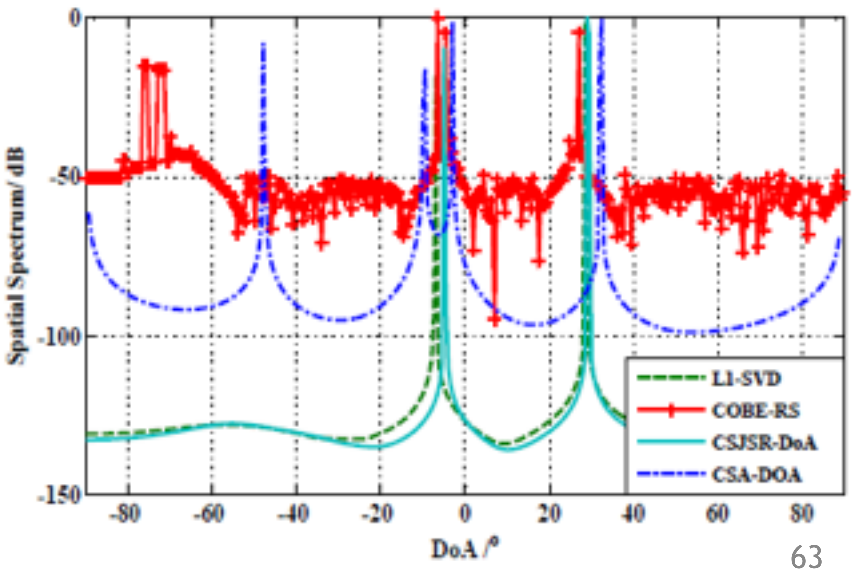
- CRB VS simulation results under different measurement number



Simulation & Experiment

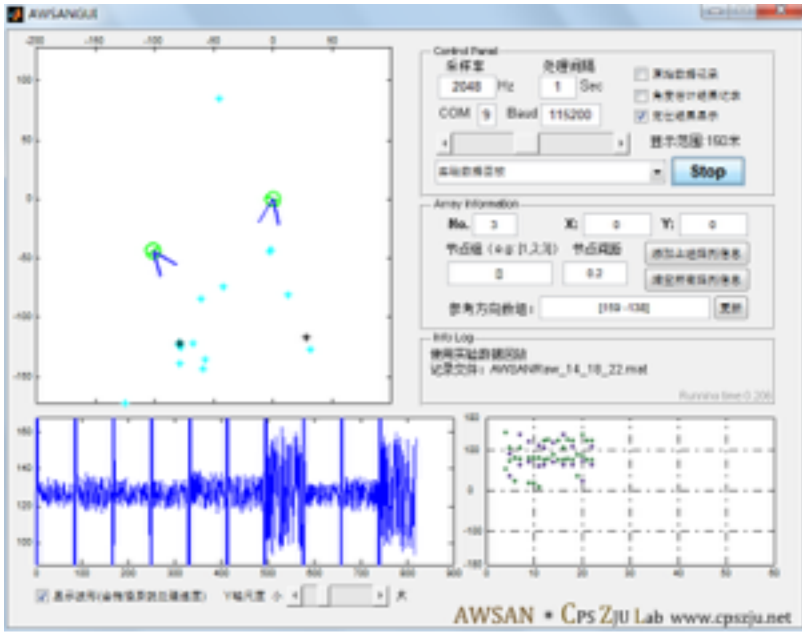
System overview

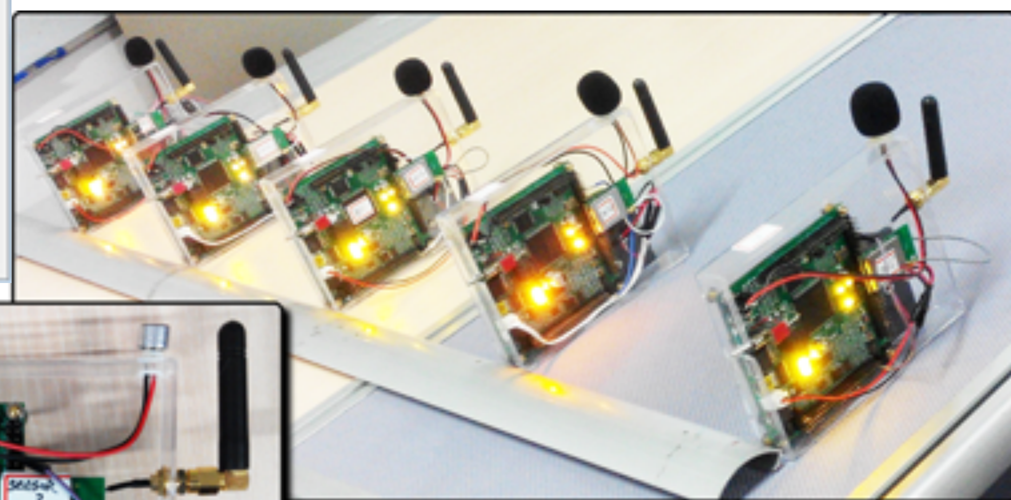


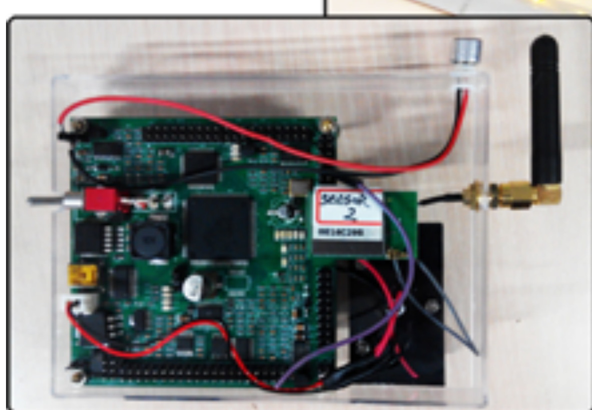


Simulation & Experiment

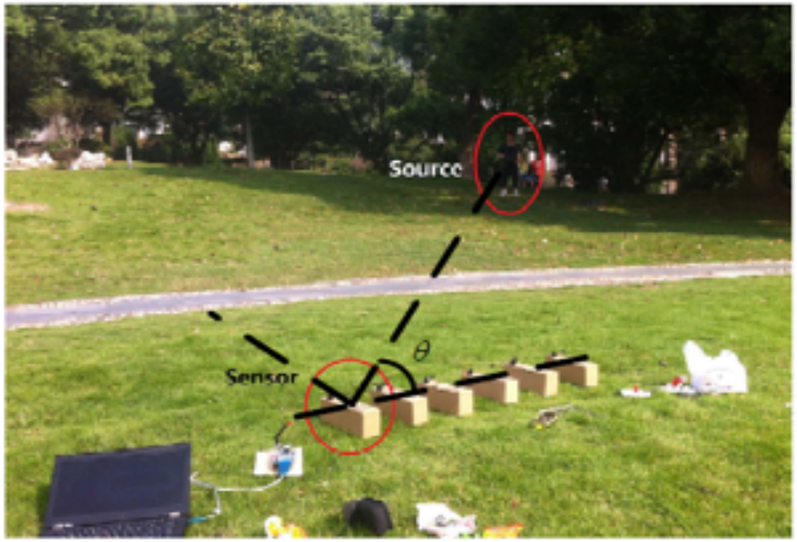
System implementation

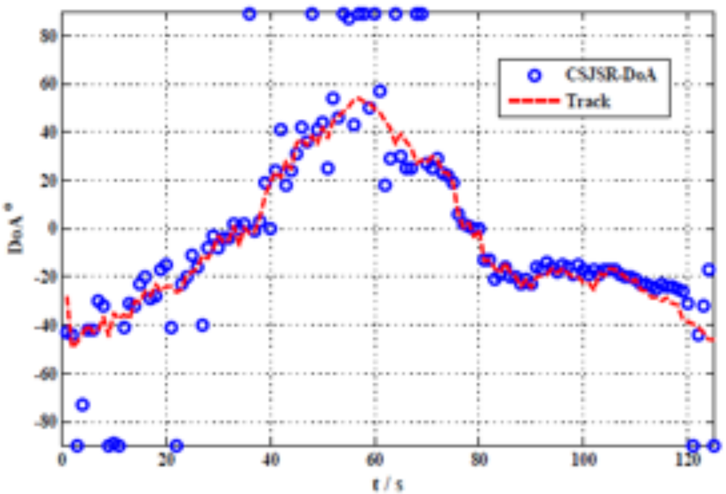






Simulation & Experiment





THE END THANK YOU!